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UNIVERSAL EDUCATION CENTRE

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TRIGONOMETRY :-

1.) $sin^2\theta + cos^2\theta = 1$

2.) $1 + tan^2\,\theta = sec^2\theta$

3.) $1 + cot^2\theta = cosec^2\theta$

4.) $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

5.) $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

6.) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \pm tanA\ tanB}$

7.) $\cot(A \pm B) = \frac{\cot A\ \ cotB \mp 1}{\cot B \pm \cot A}$

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8.) $\sin(A+B)\sin(A-B) = \begin{Bmatrix} sin^2A - sin^2B \\ or \\ cos^2B - cos^2A \end{Bmatrix}$

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9.) $\cos(A+B)\cos(A-B) = \begin{Bmatrix} cos^2A - sin^2B \\ or \\ cos^2B - sin^2A \end{Bmatrix}$

10.) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

11.) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

12.) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

13.) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

14.) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

15.) $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

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16.) cosC + cosD = 2 cos $\left(\frac{C+D}{2}\right)$ cos $\left(\frac{C-D}{2}\right)$

17.) cosC – cosD = 2 sin $\left(\frac{C+D}{2}\right)$ sin $\left(\frac{D-C}{2}\right)$ = - 2 sin $\left(\frac{C+D}{2}\right)$ sin $\left(\frac{C-D}{2}\right)$

18.) sin2A = 2 sinA cosA

19.) cos2A = cos^2A - sin^2A = 2 cos^2A – 1 = 1 – 2 sin^2A = $\frac{1-tan^2A}{1+tan^2A}$

20.) tan2A = $\frac{2\,tanA}{1-tan^2A}$

21.) 1 + cos2A = 2 cos^2A

1 – cos2A = 2 sin^2A

22.) sin3A = 3 sinA – 4 sin^3A

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23.) cos3A = 4 cos^3A - 3 cosA

24.) tan3A = $\frac{3\,tanA-tan^3A}{1-3\,tan^2A}$

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25.) sinA = 2 sin$\frac{A}{2}$ cos$\frac{A}{2}$ = $\frac{2\,tan\frac{A}{2}}{1+tan^2\frac{A}{2}}$

26.)cosA = $cos^2\frac{A}{2}$ - $sin^2\frac{A}{2}$ = $2\,cos^2\frac{A}{2}-1$ = 1 – 2 $sin^2\frac{A}{2}$ = $\frac{1-tan^2\frac{A}{2}}{1+tan^2\frac{A}{2}}$

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27.) tanA = $\frac{2\,\tan\frac{A}{2}}{1-tan^2\frac{A}{2}}$

28.) 1 + cosA = 2 $cos^2\frac{A}{2}$

1 – cosA = 2 $sin^2\frac{A}{2}$

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29.) If sinθ = sinα Then θ = nπ + $(-1)^n\alpha$, where n ϵ Z

30.) If cosθ = cosα Then θ = 2n$\pi \pm \alpha$, where n ϵ Z

31.) If tanθ = tanα Then θ = nπ + α , where n ϵ Z

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32.) $\sin\theta = 0$ Then $\theta = n\pi$, where $n \epsilon Z$

33.) $\sin\theta = 0$ Then $\theta = (2n + 1)\frac{\pi}{2}$, where $n \epsilon Z$

34.) $\tan\theta = 0$ Then $\theta = n\pi$, where $n \epsilon Z$

35.) $\sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \ldots\ldots + \sin(\alpha + \overline{n-1}\ \beta) = \dfrac{\sin\left[\alpha + \frac{(n-1)\beta}{2}\right]\sin\left\{\frac{n\beta}{2}\right\}}{\sin\left(\frac{\beta}{2}\right)}$

36.) $\cos\alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \ldots\ldots + \cos(\alpha + \overline{n-1}\ \beta) = \dfrac{\cos\left[\alpha + \frac{(n-1)\beta}{2}\right]\sin\left\{\frac{n\beta}{2}\right\}}{\sin\left(\frac{\beta}{2}\right)}$

LOGARITHM

1.) Definition :- If $a^x = n$ then $x = \log_a n$

2.) $\log_a(mn) = \log_a m + \log_a n$

3.) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

4.) $\log_a\left(\frac{m}{n}\right) = -\log_a\left(\frac{n}{m}\right)$

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5.) $\log_a m^n = n\log_a m$

6.) $\log_a 1 = 0$

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7.) $\log_a 0 = -\infty$

8.) $\log_a a = 1$

9.) $\log_b a = \dfrac{1}{\log_a b}$

10.) $\log_b a = \dfrac{\log_e a}{\log_e b}$

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11.) $a^{\log_a k} = K$

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INVERSE TRIG

Definition :- If $\sin\theta = x$ $\quad$ *Then* $\theta = \sin^{-1} x$

1.) $\sin(\sin^{-1} x) = x$ $\quad$ *or* $\quad$ $\sin^{-1}(\sin\theta) = \theta$

2.) $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x$ $\quad$ *or* $\quad$ $\sin^{-1} x = \operatorname{cosec}^{-1}\left(\frac{1}{2}\right)$

3.) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$ $\quad$ *or* $\quad$ $\cos^{-1} x = \sec^{-1}\left(\frac{1}{x}\right)$

4.) $\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x$ $\quad$ *or* $\quad$ $\tan^{-1} x = \cot^{-1}\left(\frac{1}{x}\right)$

5.) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

6.) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

7.) $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$

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8.) $\sin^{-1}(-x) = -\sin^{-1} x$

9.) $\cos^{-1}(-x) = \pi - \cos^{-1} x$

10.) $\tan^{-1}(-x) = -\tan^{-1} x$

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11.) $\cot^{-1}(-x) = \pi - \cot^{-1} x$

12.) $\sec^{-1}(-x) = \pi - \sec^{-1} x$

13.) $\csc^{-1}(-x) = -\csc^{-1} x$

14.) $\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right]$

15.) $2\sin^{-1} x = \sin^{-1}\left[2x\sqrt{1-x^2}\right]$

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16.) $3\sin^{-1} x = \sin^{-1}[3x - 4x^3]$

17.) $\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left[xy \mp \sqrt{1-x^2}\,\sqrt{1-y^2}\right]$

18.) $2\cos^{-1} x = \cos^{-1}(2x^2 - 1)$

19.) $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$

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20.) $\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left[\frac{x \pm y}{1 \mp xy}\right]$

21.) $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1-x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

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22.) $3\tan^{-1} x = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)$

23.) $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-xz}\right)$

24.) $\cot^{-1} x \pm \cot^{-1} y = \cot^{-1}\left(\frac{xy \mp 1}{y \pm x}\right)$

DIFFERENTIATION & INTEGRATION

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Fundamental Theorems of Differentiation :-

If K is a constant & u, v, w, … are functions of x. Then

1.) $\frac{d}{dx}(K) = 0$

2) $\frac{d}{dx}[K\,f(x)] = K\frac{d}{dx}\,[f(x)]$ JAYANT SHARMA (94145-37474)

3.) $\frac{d}{dx}\,[u \pm v \pm w \pm \cdots] = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx} \pm \cdots$

4.) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ ***or*** $\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + vw\frac{du}{dx} + uw\frac{dv}{dx}$

5.) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

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Fundamental Theorems of Integration :-

1.) $\int K\;f(x)dx = K\int f(x)dx$

2.) $\int[\int f_1(x) \pm f_2(x) \pm \dots\dots\dots. \pm f_n(x)]dx = \int f_1(x)dx \pm \int f_2(x)dx \pm \dots\dots\dots\dots \pm \int f_n(x)dx$

S.No.	Differentiation JAYANT SHARMA (94145-37474)	Integration
1.	$\frac{d}{dx}(c) = 0$	$\int 0\;dx = c$
2.	$\frac{d}{dx}(e^x) = e^x$	$\int e^x\,dx = e^x + c$

3.	$\frac{d}{dx}(a^x) = a^x \log a$	$\int a^x \, dx = \frac{a^x}{\log a} + c$
4.	$\frac{d}{dx}(\log x) = \frac{1}{x}$, $x \neq 0$	$\int \frac{1}{x} \, dx = \log\lvert x\rvert + c$, $where \;\; x \neq 0$
5.	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n \, dx = \frac{x^{n+1}}{n+1}$
6.	$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x \, dx = \sin x + c$
7.	$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x \, dx = -\cos x + c$
8.	$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + c$
9.	$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$	$\int \text{cosec}^2 x \, dx = -\cot x + c$
10.	$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x \, dx = \sec x + c$
11.	$\frac{d}{dx}(\text{cosec}\, x) = -\text{cosec}\, x \cot x$	$\int \text{cosec}\, x \cot x \, dx = -\text{cosec}\, x + c$
12.	$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c = -\cos^{-1} x + c$
13.	$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c = -\cos^{-1} x + c$
14.	$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c = -\cot^{-1} x + c$
15.	$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$	$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c = -\cot^{-1} x + c$
16.	$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1} x + c = -\csc^{-1} x + c$
17.	$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1} x + c = -\csc^{-1} x + c$
18.	$\frac{d}{dx}(e^{ax+b}) = ae^{ax+b}$	$\int e^{ax+b} \, dx = \frac{e^{ax+b}}{a} + c$
19.	$\frac{d}{dx}(a^{bx+c}) = ba^{bx+c} \log a$	$\int a^{bx+c} \, dx = \frac{a^{bx+c}}{b \log a} + c$
20.	$\frac{d}{dx}(\log(ax+b)) = \frac{a}{ax+b}$	$\int \frac{1}{ax+b} \, dx = \frac{1}{a}\log\lvert ax+b\rvert + c$
21.	$\frac{d}{dx}(ax+b)^n = an(ax+b)^{n-1}$	$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$
22.	$\frac{d}{dx}\sin(ax+b) = a\cos(ax+b)$	$\int \sin(ax+b) \, dx = -\frac{\cos(ax+b)}{a} + c$
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Note :- 1.) $\frac{d}{dx}\sqrt{(x)} = \frac{1}{2\sqrt{x}}$

2.) $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{2}$

Standard formulae of Integration :-

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1.) $\int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c$

2.) $\int \frac{f'(x)}{f(x)} dx = \log f(x) + c$

3.) $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$ JAYANT SHARMA (94145-37474)

4.) $\int e^{f(x)} f'(x) dx = e^{f(x)} + c$

5.) $\int a^{f(x)} f'(x) dx = \frac{a^{f(x)}}{\log a} + c$ JAYANT SHARMA (94145-37474)

Integration of $\tan x$, $\cot x$, $\sec x$ *and* $cosec\, x$:−

1.) $\int \tan x \; dx = \log(\sec x) + c$

2.) $\int \cot x \; dx = \log(\sin x) + c$

3.) $\int \sec x \; dx = \log(\sec x + tan\, x) + c = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$

4.) $\int \operatorname{cosec} x \; dx = \log(\operatorname{cosec} x - \cot x) + c = \log\left(\tan\frac{x}{2}\right) + c$

Some Standard Formulae:- JAYANT SHARMA (94145-37474)

1.) $\int \frac{1}{\sqrt{a^2 - x^2}} \; dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

2.) $\int \frac{1}{\sqrt{a^2 + x^2}} \; dx = log\left[x + \sqrt{a^2 + x^2}\right] + c$

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3.) $\int \frac{1}{\sqrt{x^2 - a^2}} \; dx = log\left[x + \sqrt{x^2 - a^2}\right] + c$

4.) $\int \frac{1}{x\sqrt{x^2 - a^2}} \; dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$

5.) $\int \frac{1}{x^2 + a^2} \; dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

6.) $\int \sqrt{a^2 - x^2} \; dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$ JAYANT SHARMA (94145-37474)

7.) $\int \sqrt{a^2 + x^2} \; dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log\left[x + \sqrt{a^2 + x^2}\right] + c$

8.) $\int \sqrt{x^2 - a^2} \; dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left[x + \sqrt{x^2 - a^2}\right] + c$

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$\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$

In above formula u is supposed to be that function which comes first in *I L A T E*, where

I → $Inverse\ function\ (\sin^{-1}x, \cos^{-1}x\ etc.)$

L → $Logarithmic\ function$

A → $Algebraic\ function\ (x^{2/3},\ 2x^2 + 3x - 5, constant)$

T → $Trigonometric\ function$

E → $Exponential\ function\ (e^x, a^x)$

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Formulae for objective questions :-

1.) $\int e^{ax}\sin bx\ dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + c \quad = \quad \frac{e^{ax}}{a^2+b^2}\sin\left(bx - \tan^{-1}\frac{b}{a}\right) + c$

2.) $\int e^{ax}\cos bx\ dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx - b\sin bx) + c \quad = \quad \frac{e^{ax}}{a^2+b^2}\cos\left(bx - \tan^{-1}\frac{b}{a}\right) + c$

3.) $\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + c$

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Partial Fraction

Condition :- Max. Power of variable in Numerator > Max. power of variable in Denominator.

Step 1 :- Factorize Denominator then →

1.	When factors in Nr. Are linear and don't Repeat	$\frac{x}{(x+3)(x-1)}$	$\frac{A}{x+3} + \frac{B}{x-1}$
2.	When max. power of variable in Nr. & Dr. are equal then add $\frac{Coeff.of\ x^n\ in\ Nr.}{Coeff.of\ x^n\ in\ Dr.}$ in Partial fraction.	$\frac{5x^2 + x + 2}{(2x-1)(3x-2)}$	$\frac{5}{6} + \frac{A}{(2x-1)} + \frac{B}{(3x-2)}$
3.	When Max. Power in Nr. > Max. power in Dr. Then divide Nr. By Dr.	$\frac{x^3 - 6x^2 + 10x - 2}{x^2 - 5x + 6}$ $= x - 1 + \frac{4-x}{x^2 - 5x + 6}$	$\frac{4-x}{x^2-5x+6} = \frac{A}{x-2} + \frac{B}{x-3}$
4.	When factors are Quadratic in Dr. don't Repeat	$\frac{x-1}{(x+1)(x^2+1)}$	$\frac{A}{x+1} + \frac{Bx+C}{(x^2+1)}$

5.	When Factors are Linear in Dr. but Repeat	$\frac{x^2}{(x-1)^3(x-2)}$	$\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{c}{(x-1)^3}+\frac{D}{(x-2)}$
6.	When factors are Quadratic in Dr. but Repeat	$\frac{2x-3}{(x-1)(x^2+1)^2}$	$\frac{A}{(x-1)}+\frac{Bx+C}{(x^2+1)}+\frac{Dx+E}{(x^2+1)^2}$
7.	When only x^2 is present in Nr. & Dr.	$\frac{x^2}{(x^2-a^2)(x^2+b^2)}$	$\frac{A}{(x^2-a^2)}+\frac{B}{(x^2+b^2)}$
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Note :-

1.) We can also use following method in 7th point

$$\frac{x^2}{(x^2-a^2)(x^2+b^2)} = \frac{A}{(x-a)}+\frac{B}{(x-b)}+\frac{Cx+D}{(x^2+b^2)}$$

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2.) To find constants we put the value of x in 1 to 6 points but in 7th point we put value of x^2.

Standard Integral :-

1.) $\int \frac{1}{x^2-a^2}\, dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+c$ $\quad$ (when $x > a$)

2.) $\int \frac{1}{a^2-x^2}\, dx = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+c$ $\quad$ (when $x < a$)

Integrals of e^x :-

1.) $\int \frac{ae^x}{b+ce^x}\, dx$	Put $e^x = t$ JAYANT SHARMA (94145-37474)
2.) $\int \frac{1}{1\pm e^x}\, dx$	Multiply & divide by e^{-x} $then\ put\ e^{-x} = t$
3.) $\int \frac{1}{e^x-e^{-x}}\, dx$ or $\int \frac{1}{(1+e^x)(1-e^{-x})}\, dx$	Write $e^{-x} = \frac{1}{e^x}$ then take L.C.M. & put $e^x = t$
4.) $\int \frac{e^{2x}-1}{e^{2x}+1}\, dx$ or $\int \frac{e^x-1}{e^x+1}\, dx$	Multiply and Divide by e^{-x} or $e^{-\frac{x}{2}}$ $resp.\ then\ put\ Denominator\ equal\ to\ 't'$ JAYANT SHARMA 94145-37474

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5.) $\int \frac{1}{\sqrt{1\pm e^x}}\,dx$ or $\int \frac{1}{\sqrt{e^x-1}}\,dx$ $\int \sqrt{1\pm e^x}\,dx$ or $\int \sqrt{e^x-1}\,dx$	Put the value inside the under root = t^2
6.) $\int \frac{1}{\sqrt{1\pm e^{2x}}}\,dx$ or $\int \frac{1}{\sqrt{e^{2x}-1}}\,dx$ or $\int \frac{e^x}{\sqrt{1\pm e^{2x}}}\,dx$ or $\int \frac{e^x}{\sqrt{e^{2x}-1}}\,dx$	Put $e^x = t$
7.) $\int \sqrt{\frac{e^x+a}{e^x-a}}\,dx$ or $\sqrt{\frac{e^x-a}{e^x+a}}\,dx$	Rationalize & then integrate.
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Trigonometrical Substitution :-

Integrals	Substitution
1.) $\sqrt{x^2+a^2}$ or $\frac{1}{\sqrt{x^2+a^2}}$	$x = a\tan\theta$
2.) $\sqrt{a^2-x^2}$ or $\frac{1}{\sqrt{a^2-x^2}}$	$x = a\sin\theta$ or $x = a\cos\theta$
3.) $\sqrt{x^2-a^2}$ or $\frac{1}{\sqrt{x^2-a^2}}$ or $\frac{1}{x\sqrt{x^2-a^2}}$	$x = a\sec\theta$
4.) $\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$	$x = a\cos 2\theta$
5.) $\sqrt{2ax-x^2}$	$x = 2a\,sin^2\theta$
6.) $\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ or $x\sqrt{\frac{a^2-x^2}{a^2+x^2}}$	$x^2 = a^2\cos 2\theta$
7.) $\sqrt{\frac{x-\alpha}{\beta-x}}$ or $\sqrt{(x-\alpha)(\beta-x)}$ or $\frac{1}{\sqrt{(x-\alpha)(\beta-x)}}$	$x = \alpha\,cos^2\theta + \beta\,sin^2\theta$ or $x-\alpha = t^2$ or $\beta - x = t^2$
8.) $\sqrt{\frac{x+a}{x}}$ or $\sqrt{\frac{x}{x+a}}$	$x = a\,tan^2\theta$

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Definite Integrals

Integration by First Principle :-

$$\int_a^b f(x)dx = \lim_{\substack{h\to 0\\ n\to\infty}} h[f(a)+f(a+h)+f(a+2h)+\ \dots\dots + f(a+\overline{n-1}h)]$$

or

$$\int_a^b f(x)dx = \lim_{\substack{h\to 0\\ n\to\infty}} h[f(a+h)+f(a+2h)+\ \dots\dots + f(a+nh)]$$ where $h = \frac{b-a}{n}$

Properties of Definite Integral :-

1.) $\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(y)dy$

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2.) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

3.) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ where a < c < b

$\int_a^b f(x)dx = \int_a^{c_1} f(x)dx + \int_{c_1}^{c_2} f(x)dx + \dots\dots + \int_{c_n}^{b} f(x)dx$ where a < c_1< c_2 <........c_n < b

4.) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Note:- If a = 0

$\int_a^b f(x)dx = \int_0^b f(b-x)dx$

5.) $\int_0^{nb} f(x)dx = n\int_0^a f(x)dx$ $\quad$ *where* $f(x)$ *is a periodic function i.e.* $f(a+x) = f(x)$

6.) $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & , \text{ when } f(x) \text{ is even. i.e. } f(-x) = f(x) \\ 0 & , \text{ when } f(x) \text{ is odd. i.e. } f(-x) = -f(x) \end{cases}$

7.) $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & , \quad f(2a-x) = f(x) \\ 0 & , \quad f(2a-x) = -f(x) \end{cases}$

An Important Integral :-

$$\int_0^{\pi/2} \log\sin x\, dx = \int_0^{\pi/2} \log\cos x\, dx = -\frac{\pi}{2}\log 2$$

$$\int_0^{\pi/2} \log\operatorname{cosec} x\, dx = \int_0^{\pi/2} \log\sec x\, dx = \frac{\pi}{2}\log 2$$

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Mensuration

1.) Area of rectangle = $l \times b$

2.) Perimeter of Rectangle = $2(l + b)$

3.) Area of square = $(side)^2$ = $(a)^2$ = $(x)^2$

4.) Perimeter of Square = 4 X side

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5.) Volume of Cuboid = $(l \times b \times h)$

6.) Total Surface Area of Cuboid = 2 X $(lb \times bh \times lh)$

7.) Volume of Cube = $(side)^3$ = $(a)^3$

8.) Total Surface Area of Cube = $6(side)^2$

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9.) Volume of Cyllinder = $\pi r^2 h$

10.) Curved Surface Area of Cyllinder = $2\pi rh$

11.) Total Surface area of Cyllinder = $2\pi r^2 + 2\pi rh$ $=$ $2\pi r(r + h)$

12.) Volume of Cone = $\frac{1}{3}\pi r^2 h$

13.) Curved Surface Area of the Cone = πrl $where \ \ l \rightarrow slant\ height,$ $l = \sqrt{r^2 + h^2}$

14.) Total Surface Area of the cone = $\pi r^2 + \pi rl$ $=$ $\pi r(r + l)$

15.) Volume of Sphere = $\frac{4}{3}\pi r^3$

16.) Surface Area of the Sphere = 4 πr^2

17.) Volume of Hemisphere = $\frac{2}{3}\pi r^3$

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18.) Total Surface Area of the Hemisphere = 3 πr^2

19.) Curved Surface Area of the Hemisphere = 2 πr^2

20.) Area of the Triangle :-

(A) When three sides are given

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, where semi-perimeter (s) = $\frac{a+b+c}{2}$

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(B) When two sides & angle b/w them is given

$$\Delta = \frac{1}{2}bc\ \sin A = \frac{1}{2}ca\ \sin B = \frac{1}{2}ab\ \sin C$$

(C) $\Delta = \frac{1}{2} \times base \times height$

(D) Area of an Equilateral Triangle = $\frac{\sqrt{3}}{4}(side)^2$

Algebra

1.) $(a+b)^2 = a^2 + 2ab + b^2$

2.) $(a-b)^2 = a^2 - 2ab + b^2$

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3.) $a^2 - b^2 = (a+b)(a-b)$

4.) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

5) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

6.) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a+b)$

7.) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$

8.) $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$

Note:- If (a+b+c) = 0 Then $a^3 + b^3 + c^3 = 3abc$

9.) $(a+b+c)^2 = (a^2 + b^2 + c^2 + 2ab + 2bc + 2ac)$

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Quadratic Equation :-

Standard equation of quadratic equation → $ax^2 + bx + c = 0$, $Where\ a \neq 0$

Quadratic Formula or Shri Dharacharya Vidhi $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here Discriminant D or $\Delta = b^2 - 4ac$

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If roots of the Quadratic Equation are α & β $then$

Sum of the Roots $(\alpha + \beta) = \frac{-b}{a}$

Product of the Roots $(\alpha\beta) = \frac{c}{a}$ **JAYANT SHARMA (94145-37474)**

Note :-

1.) If roots are real & equal then D = 0

2.) if roots are real & not equal then D > 0

3.) If roots are real then D ≥ 0

4.) If D < 0 , then roots are not real. **JAYANT SHARMA (94145-37474)**

Cubic Polynomial

Standard equation of Cubic Polynomial $\rightarrow$ $ax^3 + bx^2 + cx + d = 0$, $where\ a \neq 0$

If $\alpha, \ \beta, \ \gamma$ $are\ the\ roots\ \ then$

$\alpha + \beta + \gamma = -\frac{b}{a}$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$ **JAYANT SHARMA (94145-37474)**

$\alpha\beta\gamma = -\frac{d}{a}$

Arithmetic Progression (A.P) :-

If first term is a, common difference is d and last term is l then

Sum of n terms $\rightarrow$ $s_n = \begin{Bmatrix} \frac{n}{2}\{2a + (n-1)d\} \\ or \\ \frac{n}{2}(a + l) \end{Bmatrix}$ **JAYANT SHARMA (94145-37474)**

1.) $\sum n = 1 + 2 + 3 + \ldots\ldots\ldots\ldots + n = \frac{n(n+1)}{2}$

2.) $\sum n^2 = 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

3.) $\sum n^3 = 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ **JAYANT SHARMA (94145-37474)**

Note :- $n^{th}\ term\ of\ A.P.$ $\rightarrow$ $a_n = a + (n-1)d$

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Geometric Progression

If a is the first term of a G.P. and r its common ratio, then its nth term , $t_n = ar^{n-1}$

The sum S_n of the first n terms of such a G.P. is given by

$$S_n = \begin{cases} \frac{a(r^r-1)}{r-1} & if \ r \neq 1 \\ na & if \ r = 1 \end{cases}$$

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Note :- 1.) The Geometric mean G of two positive numbers a and b is given by $\sqrt{ab}$.

2.) The Arithmetic mean G of two positive numbers a and b is given by $\frac{a+b}{2}$

Complex Numbers

A number of the form $a + ib$ where a,b $\in R$, the set of the real numbers, and $i = \sqrt{-1}$, is called a complex number.

1.) If z = $a + ib$, then the real part of z is denoted by Re(z) & imaginary part is Im(z).

2.) Conjugate of complex Number :-

If z = $a + ib$ then conjugate of z is $\bar{z} = a - ib$.

If z = $a - ib$ then conjugate of z is $\bar{z} = a + ib$.

Properties of Conjugate of a Complex Number

1.) $z_1 = z_2 \Leftrightarrow \overline{z_1} = \overline{z_2}$

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2.) $\overline{(\bar{z})} = z$

3.) z$\bar{z} = [Re(z)]^2 + [Im(z)]^2$

4.) $\overline{z_1 \pm z_2} = \overline{z_1} + \overline{z_2}$

5.) $\overline{z_1 z_2} = \overline{z_1 z_2}$

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6.) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

Modulus of a complex Number

If z = $a + ib$ then modulus of z is $|z| = \sqrt{a^2 + b^2}$

Straight Lines

Let A(x_1, y_1) *and* $B(x_2, y_2)$, $(x_1 \neq x_2)$ *be any* two points. The slope of the line joining A and B is defined as

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$$

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Where θ is the angle which the line makes with the positive direction of the $x - axis$.

Standard Forms of the Equation of a Line:-

1.) parallel to the $x - axis$. is $y = b$ $(where\ b\ is\ y - intercept)$

Parallel to the $y - axis$ *is* $x = a$ $(where\ a\ is\ x - intercept)$

Equation of $x - axis$ *is* $y = 0$ and Equation of $y - axis$ *is* $x = 0$

2.) Slope-Intercept form:-

$$y = mx + c \ , \qquad (c\ is\ the\ intercept\ on\ y - axis)$$

3.) Point – Slope form :→ $y - y_1 = m(x - x_1)$

4.) Two-Point form :- $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

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5.) Intercept- form :- $\frac{x}{a} + \frac{y}{b} = 1$

6.) Normal form :- $x\cos\alpha + y\sin\alpha = p$

General form of the equation of a line is $Ax + By + C = 0$

1.) The Slope of the Line is $-\frac{A}{B}$

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2.) The Intercept on the x-axis is $-\frac{C}{A}$ & The Intercept on the y-axis is $-\frac{C}{B}$

3.) $\cos\alpha = \pm\frac{|a|}{\sqrt{a^2+b^2}}$, $\sin\alpha = \pm\frac{|b|}{\sqrt{a^2+b^2}}$, $p = \frac{|c|}{\sqrt{a^2+b^2}}$

4.) Length of the perpendicular from (x_1, y_1), on the line $Ax + By + C = 0$

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{a^2+b^2}}\right|$$

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5.) If two lines $(a_1x + b_1y + c_1 = 0)$ *and* $(a_2x + b_2y + c_2 = 0)$ are parallel then their slopes are equal $(m_1 = m_2)$

6.) If two lines $(a_1x + b_1y + c_1 = 0)$ *and* $(a_2x + b_2y + c_2 = 0)$ are perpendicular to each other then the product of their slopes is -1 $(m_1m_2 = -1)$

7.) angle θ between them at their point of intersection is $\tan\theta = \pm \frac{m_1-m_2}{1+m_1m_2}$

8.) distance between two parallel lines is $d = \frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$

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Circles

1.) Equation of a circle with centre(h, k) and radius r is :-

$$(x-h)^2 + (y-k)^2 = r^2$$

2.) Equation of a circle with centre(0 , 0) and radius r is :-

$$(x)^2 + (y)^2 = r^2$$

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3.) General equation of a circle :-

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad , \qquad where\ \ g, f\ and\ c\ are\ constants.$$

Centre of this circle is (-g, -f). And r= $\sqrt{g^2 + f^2 - c}$, $(g^2 + f^2 \geq c)$

4.)Length of x-intercept made by the circle is $2\sqrt{g^2 - c}$, $if\ \ (g^2 - c \geq 0)$

Length of y-intercept made by the circle is $2\sqrt{f^2 - c}$, $if\ \ (f^2 - c \geq 0)$

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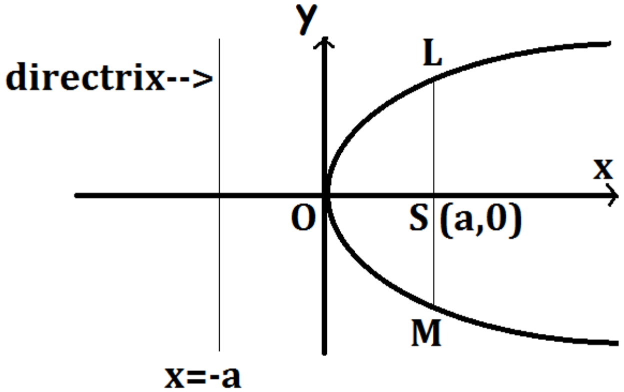
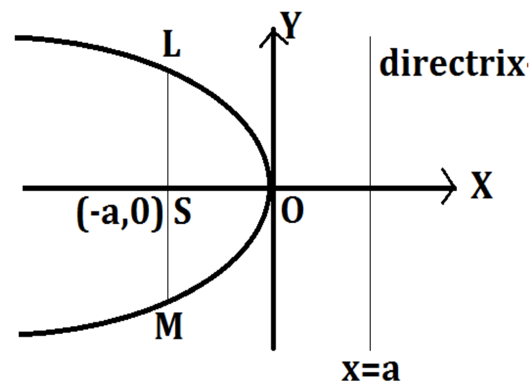
Parabola



eq. of parabola $y^2 = 4ax$



eq. of parabola $y^2 = -4ax$

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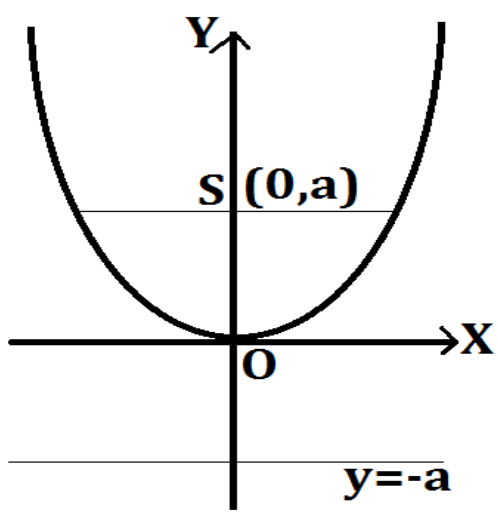
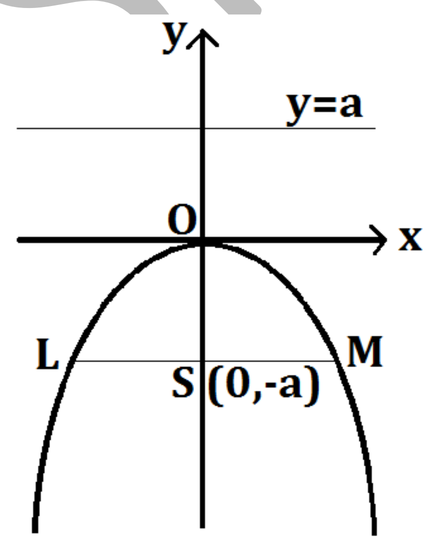


eq. of parabola $x^2 = 4ay$



eq. of parabola $x^2 = -4ay$

Parabola	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
1.) Vertex	(0 , 0)	(0 , 0)	(0 , 0)	(0 , 0)
2.) Focus	(a , 0)	(-a , 0)	(0 , a)	(0 , -a)
3.) Eq. of axis	Y = 0	Y = 0	X = 0	X = 0
4.) Eq. of directrix	X= -a	X = a	Y = -a	Y = a
5.) Eq. of Latus Rectum	X = a	X = -a	Y = a	Y = -a
6.) Length of L.R.	4a	4a	4a	4a

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Ellipse :-

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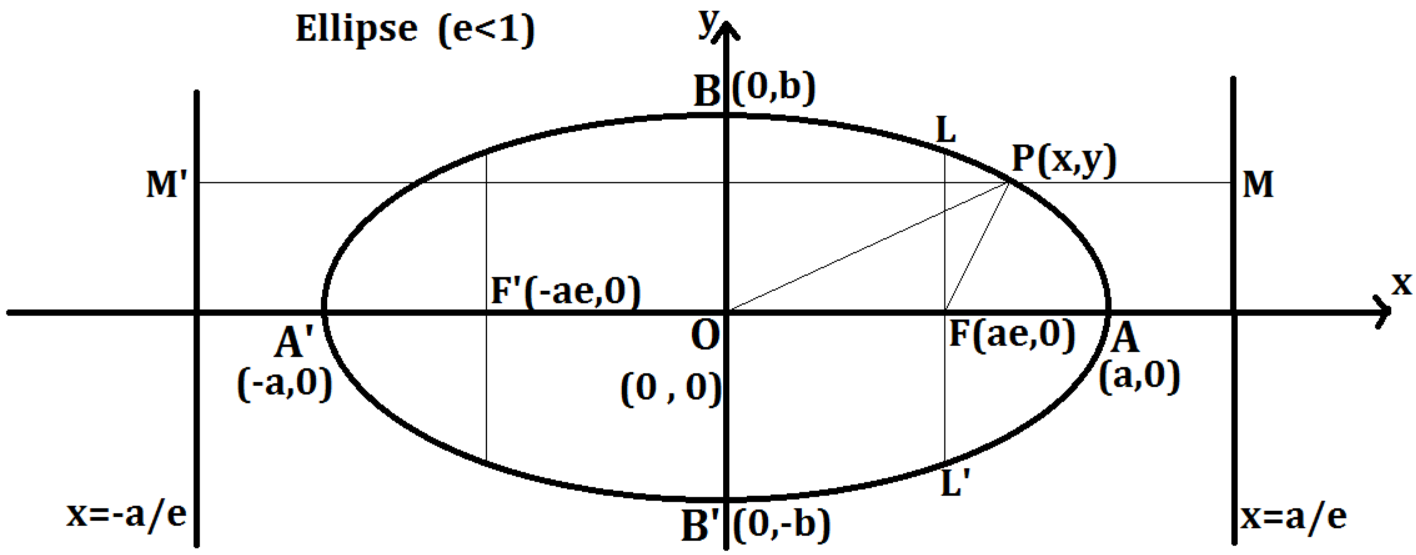

$$Eq.\,of\ Ellipse\ \ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

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Eq. of Ellipse	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ ,\ \ a>b$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ ,\ \ a<b$
Coordinates of the centre	(0 , 0)	(0 , 0)
Coordinates of the vertices	(a,0) and (-a,0)	(0, b) and (0, b)
Coordinates of the foci	(ae, 0) and (-ae, 0)	(0, be) and (0, -be)
Length of the major axis	2a	2b
Length of the minor axis	2b	2a
Eq. of the major axis	Y=0	x=0
Eq. of the minor axis	X=0	y=0
Eqs. Of the Directrices	$x=\frac{a}{e}\ and\ \ x=-\frac{a}{e}$	$y=\frac{b}{e}\ and\ \ y=-\frac{b}{e}$
Eccentricity	$e=\sqrt{1-\frac{b^2}{a^2}}$	$e=\sqrt{1-\frac{a^2}{b^2}}$
Length of the Latus Rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Focal distances of a point (x,y)	a$\pm ex$	b$\pm ey$

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Hyerbola :-

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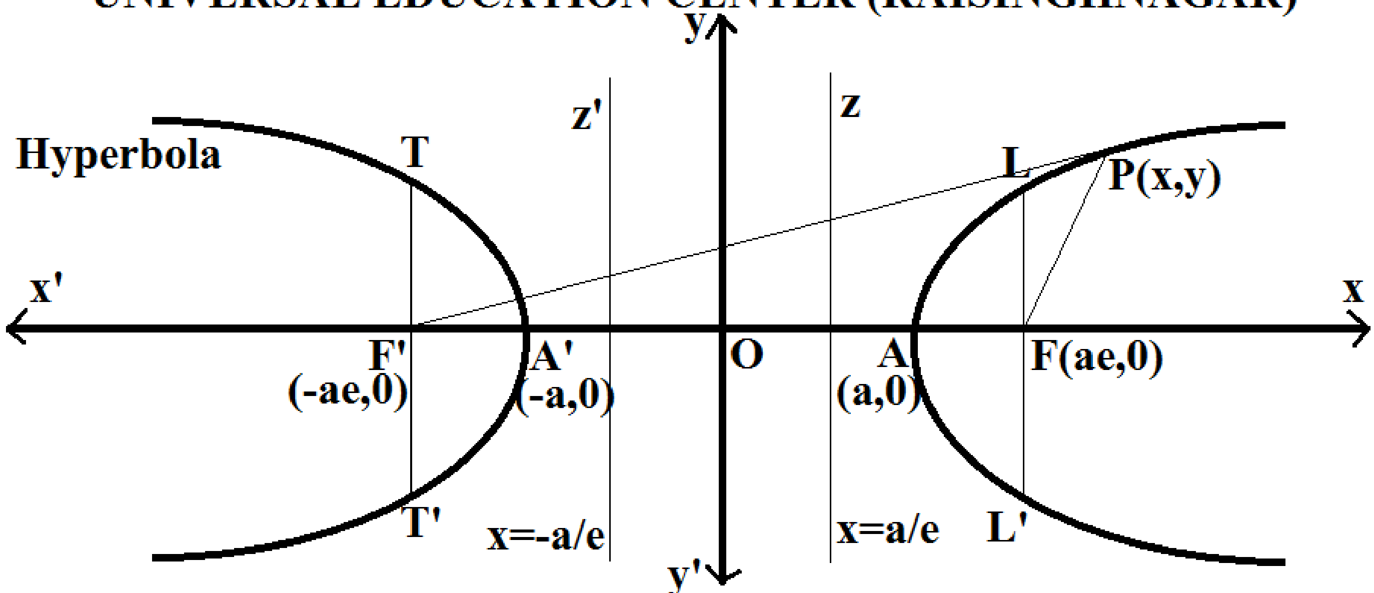

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$$Eq.of\ Hyperbola = \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	Conjugate Hyperbola $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Coordinates of the centre	(0, 0)	(0, 0)
Coordinates of the vertices	(a, 0) and (-a, 0)	(0, b) and (0, -b)
Coordinates of foci	$(\pm ae, 0)$	$(0, \pm be)$
Length of the transverse axis	2a	2b
Length of the Conjugate axis	2b	2a
Equations of the directrices	$x = \pm\frac{a}{e}$	$y = \pm\frac{b}{e}$
Eccentricity	$e = \sqrt{\frac{a^2+b^2}{a^2}}$ $or, b^2 = a^2(e^2-1)$	$e = \sqrt{\frac{a^2+b^2}{b^2}}$ $or,$ $a^2 = b^2(e^2-1)$

Length of Latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
Equation of the transverse axis	Y=0	X=0
Equation of the conjugate axis	X=0	Y=0
Focal distances	$ex \pm a$	$ey \pm b$

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