AIMS INSTITUTE

A PREMIER INSTITUTE OF MATHEMATICS FOR IIT, XI & XII SINCE 1998

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First Mock Board Exam

By: Sandeep Shishodia

Duration: $2\frac{1}{2}$ **Hours** | **Class XII** | **Max. Mark 100.**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.*
3. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
4. *There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six mark each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculators is not permitted. You may ask for logarithmic tables, if required.*

Section-A

(*1 mark each*)

1. Write the antiderivative of $\frac{2^x}{1+4^x}$ w.r.t. 'x'.

2. Write the principal value branch of $arc\ \sin x$.

3. *If* $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ *and* $|3A| = \lambda |A|$ *then* find λ.

4. Find the angle between line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-1}{6}$ and the plane $10x + 2y - 11z = 3$.

5. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on vector $7\hat{i} - \hat{j} + 8\hat{k}$.

6. Write the number of all binary operation on set A = {a, b, c} to itself.

7. If the matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is skew symmetric matrix then find the value of *a*, *b* and *c*.

8. Find the value of a such that the line $\frac{x-2}{9} = \frac{y-1}{a} = \frac{z+3}{-6}$ is perpendicular to the plane $3x - y - 2z = 7$.

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9. Find the value of $\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

10. Differentiate $\frac{\sin \lfloor x}{y^{\circ}} w.r.t.\ 'y'.$

Section-B

(*4 marks each*)

11. Evaluate $\int x^x.(1+\log x)\ dx$

OR

Evaluate: $\int \frac{x^2}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}\ dx$

12. Let $A = N \times N$. Let * be binary operation on A defined by $(a,b)*(c,d)=(ad+bc,bd)$. Then find the identity element of $(A,*)$.Is $(A,*)$ commutative?

13. Solve for x, $2\tan^{-1}(\cos\ x)=\tan^{-1}(2\operatorname{cosec}\ x)$.

OR

Prove that: $\tan\left(\frac{\text{®}}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\text{®}}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}.$

14. Prove by using properties of determinants:

$$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3.$$

15. For what value of a and b, the function defined as:

$$f(x)=\begin{cases} 3ax+b & ;\ if\ \ x<1 \\ 11 & ;\ if\ \ x=1 \\ 5ax-2b & ;\ if\ \ x>1 \end{cases}$$ is continuous at $x=1$.

Mobile phone is a necessary evil, comment on it

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16. If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that $\frac{\left[1+(y')^2\right]^{3/2}}{y''}$ is a constant and free from a and b.

OR

If $\sqrt{1-x^{2n}}+\sqrt{1-y^{2n}}=a(x^n-y^n)$, prove that $\frac{dy}{dx}=\frac{x^n}{y^n}\sqrt{\frac{1-y^{2n}}{1-x^{2n}}}$.

17. Find the interval in which the function *f* given by $f(x)=\sin^4 x+\cos^4 x$ is strictly increasing and strictly decreasing $0<x<\frac{\text{®}}{2}$.

18. Solve the differential equation: $\frac{dy}{dx}-\frac{1}{x}.y=2x^2$. *Social websites are consuming precious time and money of the students, do you agree/disagree, give two lines in support of your answer.*

19. Solve: $(x^3+x^2+x+1)\frac{dy}{dx}=2x^2+x$.

20. There are three bags each bag contains 4red & 5black, 7red & 5black and 4red & 6black. One bag is selected at random and two black balls are drawn from the bag and distributed one-one into other bags then a ball is drawn, and that is black find the probability that the two black balls were drawn from the bag which now contained 7 balls.

21. $\vec{a},\vec{b},\vec{c}$ are the unit vectors. Suppose $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\text{®}}{6}$, prove that $\vec{a}=\pm 2(\vec{b}\times\vec{c})$.

OR

If $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-5\hat{k}$, then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are orthogonal.

22. If the straight lines $\frac{x+1}{2}=\frac{y+1}{5}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{5}$ are coplanar, find the equation of plane containing these two lines.

Section-C

(6 marks each)

23. Evaluate $\int_1^4 (x^2-x)dx$ as the limit of a sum.

24. A bag contains 4 balls. Two balls are drawn at random, and are found to be blue. What is the probability that 50% balls were blue in colour, in that bag?

25. If $A=\begin{bmatrix}2 & 1 & 3\\ 4 & -1 & 0\\ -7 & 2 & 1\end{bmatrix}$ find A^{-1}, and hence solve the following system of equations:

$$\begin{aligned} 2x+y+3z&=3\\ 4x-y\qquad&=3\\ -7x+2y+z&=2. \end{aligned}$$

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26. Using integration find the area of the region included between the curves

$y = x^2 + 1, y = x, x = 0$ *and* $y = 2.$

OR

Using definite integration, find the area of the region: $\left\{(x, y) : |x-1| \le y \le \sqrt{5-x^2}\right\}.$

27. Find the distance of the point (1, -2, 3) from the plane x – y + z = 5 measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}.$

OR

Find foot, length and equation of the perpendicular drawn from the point (1, 1, 1) to the plane $\vec{r}.(2\hat{i} - 2\hat{j} + 4\hat{k}) + 5 = 0$

28. A window of fixed perimeter (including the base of the triangle) is in the form of a rectangle surmounted by an equilateral triangle. The triangular portion is filled with colored glass while the rectangular part is filled with clear glass. The coloured glass stops 30% light fall on it while clear glass only 1%. What is the ratio of the sides of the rectangle so that the window transmits the maximum light? *Use sunlight to save electricity and serve the nation directly comments on it.*

29. There are two factories located one a place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirement of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

To / From	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum? What will be the minimum transportation cost? *How we can help our parents/nation to save money by using easy/public transportation*.

All the best

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