HALF SYLLABUS TEST XII - 03

By : OP GUPTA
(+91-9650 350 480)

Time Allowed : 120 Minutes **Max. Marks : 75**

[SECTION – A]

Q01. The total cost associated with provision of free mid-day meals to x students of a school in primary classes is given by

$$C(x) = 0.005x^3 - 0.02x^2 + 30x + 50$$

If the marginal cost is given by rate of change $\frac{dC}{dx}$ of total cost, write the marginal cost of food for 300 students. What value is shown here?

Q02. If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then write the value of (x + y).

Q03. If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, then write the value of x.

Q04. If A is a square matrix of order 3 such that |adj A| = 64, find the value of |A|.

Q05. Write the value of the following :

$\tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{a-b}{a+b}\right)$.

Q06. Write the principal value of $\left[\cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\left(-\frac{1}{2}\right)\right]$.

Q07. Write the projection of the vector $7\hat{i} + \hat{j} - 4\hat{k}$ on the vector $2\hat{i} + 6\hat{j} + 3\hat{k}$.

Q08. Write the value of λ so that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other.

Q09. Write the degree of the differential equation :

$\left(\frac{dy}{dx}\right)^4 + 3y\frac{d^2y}{dx^2} = 0$.

Q10. Check whether Mean Value Theorem is applicable for the function $f(x) = |x|$ on [–1, 1].

[SECTION – B]

Q11. If $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, \text{ when } x < 0 \\ a, \text{ when } x = 0 \\ \frac{\sqrt{x}}{\left(\sqrt{16+\sqrt{x}}\right) - 4}, \text{ when } x > 0 \end{cases}$ and f is continuous at x = 0, find the value of a.

Q12. Evaluate : $\int \frac{(3\sin x - 2)\cos x}{5 - \cos^2 x - 4\sin x} dx$.

(OR) Evaluate : $\int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right) dx$.

Q13. Evaluate : $\int_0^{\pi/4} \log(1 + \tan x) dx$.

Q14. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$ then, prove that $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$.

(OR) If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

Q15. Using properties of determinants, prove that :

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$$

Q16. Prove that : $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$.

(OR) Solve for x : $\tan^{-1}3x + \tan^{-1}2x = \frac{\pi}{4}$.

Q17. If $y = \log[x + \sqrt{x^2 + a^2}]$, then prove that $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

Q18. Evaluate : $\int \frac{3x+5}{x^3 - x^2 - x + 1}dx$.

Q19. Dot product of a vector with vectors $\hat{i} - \hat{j} + \hat{k}$, $2\hat{i} + \hat{j} - 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$ are respectively 4, 0 and 2. Find the vector.

[SECTION – C]

Q20. If a young man drives his scooter at 25kmph, he has to spend ₹2 per kilometer on petrol. If he drives the scooter at a speed of 40kmph, it produces more air pollution and increases his expenditure on petrol to ₹5 per kilometer. He has a maximum of ₹100 to spend on petrol and wishes to travel a maximum distance in one hour time with less pollution. Express this problem as an LPP and solve it graphically. What value do you find here?

Q21. Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of ₹x, ₹y and ₹z respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with a total prize money of ₹37000 and the second institution decided to award respectively 5, 3 and 4 employees with a total prize money of ₹47000. If all the three prizes per person together amount to ₹12000, then using matrix method find the value of x, y and z.
What values are described in this question?

Q22. Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$. Also find the equation of corresponding normal.

(OR) Prove that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

Q23. Using integration, find the area of the region enclosed by the curves $y^2 = 4x$ and $y = x$.

Q24. Find the particular solution of the following differential equation given that y = 0 when x = 1 :
$(x^2 + xy)dy = (x^2 + y^2)dx$.

www.theOPGupta.com/

ANSWERS *Of* HST XII – 03 [2014 - 15]

Q01. ₹1368
Q02. (x + y) = 3 + 3 = 6.
Q03. 1.
Q04. ± 8.
Q05. $\frac{\pi}{4}$.
Q06. $\frac{5\pi}{6}$.
Q07. $\frac{8}{7}$.
Q08. $\frac{5}{2}$.
Q09. Degree : 1.
Q10. Not applicable as $f(x)$ is not differentiable at x= $0 \in [-1, 1]$.
Q11. Value of a = 8.
Q12. $3\log|\sin x - 2| - \frac{4}{\sin x - 2} + C$. **(OR)** $-\frac{1}{2}e^{2x}\cot x + C$.
Q13. $\frac{\pi}{8}\log 2$.
Q14. **(OR)** $(\sin x)^x\left[x\cot x + \log\sin x\right] + \frac{1}{2\sqrt{x - x^2}}$.
Q16. **(OR)** $x = \frac{1}{6}$.
Q18. $\log\left|\frac{x+1}{x-1}\right| - \frac{4}{x-1} + C$.
Q19. $2\hat{i} - \hat{j} + \hat{k}$.
Q20. Maximum distance = 30 km at $\left(\frac{50}{3}, \frac{40}{3}\right)$
Q21. Values of x, y and z are respectively ₹4000, ₹5000 and ₹3000.
Q22. Equation of the tangent : $48x - 24y = 23$ and equation of normal : $48x + 96y = 113$.
Q23. $\frac{8}{3}$ Sq.Units.
Q24. $y = x\left[\log|x| - 2\log|y - x|\right]$