

HALF SYLLABUS TEST XII – 02

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Max. Marks : 70

Time Allowed : 2 Hours

Section A

- Q01.** An edge of variable cube is increasing at the rate of 3cm/s. How fast is the volume of the cube increasing when the edge is 10cm long?
- Q02.** If ‘ \wedge ’ is a binary operation which is defined as “ $a \wedge b = a^2 + 2b$ ” then, determine the value of $3 \wedge 2$.
- Q03.** Evaluate : $\int \frac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}} dx$.
- Q04.** Write the value of $\int_{-\pi/2}^{\pi/2} \log \left| \frac{2 - \sin x}{2 + \sin x} \right| dx$.
- Q05.** Write the integrating factor of differential equation : $(1 + y^2) dx = (\tan^{-1}y - x) dy$.
- Q06.** If $x \in \mathbb{R}$, $0 \leq x \leq \frac{\pi}{2}$, and $\begin{vmatrix} 2\sin x & -1 \\ 1 & \sin x \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ -4 & \sin x \end{vmatrix}$, then find the values of x .

Section B

- Q07.** Find the interval in which the function $x^4 - 8x^3 + 22x^2 - 24x + 21$ is increasing or decreasing.
- OR** A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of $5m^3/hr$. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4m.
- Q08.** If $x = a(t - \sin t)$, $y = a(1 - \cos t)$, find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$.
- OR** Differentiate $x^{x \cos x} + \frac{x^2 + 1}{x^2 - 1}$ w.r.t. x .
- Q09.** Evaluate $\int_1^3 (x^2 + x) dx$ as the limit of a sum.
- Q10.** Evaluate : $\int \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx$.
- Q11.** Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be defined as $f(n) = \begin{cases} \frac{n+1}{2}, & \text{when } n \text{ is odd} \\ \frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$ for all $n \in \mathbb{N}$.
- State whether the function f is bijective. Justify your answer.
- Q12.** Solve : $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$.
- Q13.** For what value of k , $f(x) = \begin{cases} \frac{\log(1+ax) - \log(1-bx)}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$?

Section C

- Q14.** Prove that the volume of largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.
- Q15.** Evaluate : $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$.
- OR** Evaluate : $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$.
- Q16.** Using properties of determinants, prove that : $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$.
- Q17.** Use transformations to find inverse of $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.
- Q18.** Prove that : $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x}) dx = \sqrt{2} \pi$.
- Q19.** Sketch the region common to $x^2 + y^2 = 16$ and $x^2 = 6y$. Also find the area of the enclosed region.

ANSWERS Of HST XII – 02 [2014 - 15]

Q01. $900\text{cm}^3/\text{s}$

Q02. 13

Q03. $\frac{x^3}{3} + C.$

Q04. 0.

Q05. $e^{\tan^{-1}y}.$

Q06. $\frac{\pi}{6}, \frac{\pi}{2}.$

Q07. Increasing in $[1, 2] \cup [3, \infty)$ and Decreasing in $(-\infty, 1] \cup [2, 3].$

OR $\frac{35}{88} \text{m/hr}.$

Q08. $-\frac{1}{a}.$ OR $x^x \cos x (\cos x + \cos x \log x - x \sin x \log x) - \frac{4x}{(x^2 - 1)^2}.$

Q09. $\frac{38}{3}.$

Q10. $\log \left| \frac{1 + \sin x}{2 + \sin x} \right| + C.$

Q11. Function f is not bijective as f is onto but not one-one.

Q12. $x = 0.$

Q13. $k = a + b$

Q15. $\left(\frac{\pi}{2}\right)^2.$ OR $\frac{\pi^2}{2ab}.$

Q17. $\begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}.$

Q19. $\frac{4}{3}[\sqrt{3} + 4\pi] \text{ Sq. units}.$

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