SUBJECTIVE TEST SERIES XI – 14

Max. Marks : 100 **Based on Ch 01 To 08** **Time Allowed : 2.5 Hours**

General Instructions :

(i) The question paper consists of three parts A, B and C. Each question of each part is compulsory.
(ii) Section A (Very Short Answer Type) consists of 06 questions of 1 mark each.
(iii) Section B (Short Answer Type) consists of 13 questions of 4 marks each.
(iv) Section C (Long Answer Type) consists of 7 questions of 6 marks each.
(v) Use of calculator is not permitted.

SECTION – A

(This section contains 6 questions of **one marks** each.)

Q01. If A and B are two disjoint sets containing 5 and 8 elements respectively, then determine the number of elements in "A or B".

Q02. Given $f(x) = |x - 3|$. Find the domain and range of the function f.

Q03. Find the principal solutions of the equation $\tan x = -\frac{1}{\sqrt{3}}$.

Q04. If P(n) is a statement "12n + 5 is a multiple of 13", check if P(2) is true or false.

Write the correct alternative in Q05 & Q06 :

Q05. If $x + i y = 2 - i\sqrt{3}$, then the value of $x^2 + y^2$ is

(A) 7 (B) $\sqrt{7}$ (C) ± 7 (D) None

Q06. Value(s) of x in $8x - 2 \le 5x + 1$ s.t. $x \in N$ is

(A) {1} (B) {..., –1, 0, 1} (C) {1, 2, 3, ...} (D) {..., –1, 0, 1, ...}

SECTION – B

(This section contains 13 questions of **four marks** each.)

Q07. Let A = {1, 2, 3, 5, 6} and B = {2, 3, 4, 6, 7, 8} are sets associated with U = {1, 2, 3, ..., 10}. State and verify De-Morgan's Laws for sets A and B.

Q08. If $f : R \to R$ is defined by $f(x) = \frac{x}{x^2 + 1}$, find $f[f(2)]$.

Q09. Prove that : sin 3x + sin 2x – sin x = 4 sin x cos (x/2) cos (3x/2).

Q10. Find the general solution of the equation : $\sin 3\theta + \cos 2\theta = 0$.

Q11. Write the relation R = {(x, x + 5) : $x \in \{0,1,2,3,4,5\}$} in roster from and determine its domain and range.

Q12. Prove by using induction, $1.3 + 3.5 + 5.7 + ... + (2n-1)(2n+1) = \frac{n(4n^2 + 6n - 1)}{3}$, $n \in N$.

Q13. Write the complex number $\frac{1+2i}{1-3i}$ in its polar form.

Q14. **(i)** How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE?
(ii) A bag contains six black and four red balls. Determine the number of ways in which two black and three red balls can be selected.

Q15. A committee of seven has to be formed from nine boys and four girls. In how many ways can this be done when the committee consists of
(i) exactly three girls? **(ii)** at least three girls?

Q16. Find $(a + b)^4 - (a - b)^4$. Hence evaluate $[\sqrt{3} + \sqrt{2}]^4 - [\sqrt{3} - \sqrt{2}]^4$.

Q17. Find the term independent of x in the binomial expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^6$.

Q18. Prove that : $\cos 52^\circ + \cos 68^\circ + \cos 172^\circ = 0$.

Q19. Success (S) in life is a linear function of values (V) in life which we have acquired. Represent S as a function of V. Do you agree? Mention two life skills.

SECTION – C

(This section contains 7 questions of **six marks** each.)

Q20. In an examination, 80% students passed in Mathematics, 72% passed in Science and 13% failed in both the subjects. If 312 students passed in both the subjects, find the total number of students who appeared in the examination. Copying in examination is dishonesty. What other views you have on adverse impact of copying in one's life?

Q21. In triangle ABC, prove that : $\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}$.

Q22. Find the square roots of $-4-3i$.

Q23. Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If these words are written as in a dictionary, what will be the 49^{th} word?

Q24. Find a, b and n in the expansion of $(a+b)^n$ if the first three terms of the expansion are 729, 7290 and 30375.

Q25. Solve the following system of inequalities graphically :
$2x+y\geq 8,\ x+2y\geq 10,\ x\geq 0,\ y\geq 0$.

Q26. Prove by using principle of mathematical induction that $\frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}$ is a natural number.

Q01. Since A and B are two disjoint sets, so $A \cap B = \phi$. Therefore number of elements in "A or B" i.e., $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 13$.

Q02. Domain : $x \in R$ and range : $y \in [0, \infty)$ where $y = f(x)$.

Q03. $\frac{5\pi}{6}, \frac{11\pi}{6}$. **Q04.** False. **Q05.** (A) 7 **Q06.** (A) {1}

Q07. **De-Morgan's laws :** 'not (A and B)' is the same as '(not A) or (not B)' i.e., $(A \cap B)' = A' \cup B'$. Also, 'not (A or B)' is the same as '(not A) and (not B)' i.e., $(A \cup B)' = A' \cap B'$.

Now $A' = \{4,7,8,9,10\}$, $B' = \{1,5,9,10\}$, $A \cap B = \{2,3,6\}$, $(A \cap B)' = \{1,4,5,7,8,9,10\}$ ...(i)

$\therefore A' \cup B' = \{1,4,5,7,8,9,10\} = (A \cap B)'$ [By using (i)

Similarly, $A \cup B = \{1,2,3,4,5,6,7,8\}$, $(A \cup B)' = \{9,10\}$ ...(ii)

$A' \cap B' = \{9,10\} = (A \cup B)'$ [By using (ii). Hence De-Morgan's laws are verified.

Q08. We have $f(2) = \frac{2}{2^2+1} = \frac{2}{5}$ so, $f[f(2)] = f\left(\frac{2}{5}\right) = \frac{2/5}{(2/5)^2+1} = \frac{10}{29}$.

Q09. LHS : sin 3x + sin 2x – sin x = $2\sin\frac{3x+2x}{2}\cos\frac{3x-2x}{2} - 2\sin\frac{x}{2}\cos\frac{x}{2}$

$$= 2\cos\frac{x}{2}\left(\sin\frac{5x}{2} - \sin\frac{x}{2}\right) = 2\cos\frac{x}{2}\left(2\cos\frac{\frac{5x}{2}+\frac{x}{2}}{2}\sin\frac{\frac{5x}{2}-\frac{x}{2}}{2}\right)$$

$$= 2\cos\frac{x}{2}\left(2\cos\frac{3x}{2}\sin x\right) = 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2} = \text{RHS}.$$

Q10. Given $\sin 3\theta + \cos 2\theta = 0$ $\Rightarrow \cos 2\theta = -\sin 3\theta$ $\Rightarrow \cos 2\theta = \cos\left(\frac{\pi}{2} + 3\theta\right)$

$\Rightarrow 2\theta = 2n\pi \pm \left(\frac{\pi}{2} + 3\theta\right), n \in Z$ $\Rightarrow 2\theta = 2n\pi + \left(\frac{\pi}{2} + 3\theta\right), 2\theta = 2n\pi - \left(\frac{\pi}{2} + 3\theta\right), n \in Z$

$\Rightarrow \theta = -\left(2n\pi + \frac{\pi}{2}\right), \theta = \frac{1}{5}\left(2n\pi - \frac{\pi}{2}\right), n \in Z$ $\therefore \theta = -\frac{\pi}{2}(4n+1), \frac{\pi}{10}(4n-1); n \in Z$

or, $\therefore \theta = -\frac{\pi}{2}(2m+1), \frac{\pi}{10}(2m-1); m \in Z$.

Q11. Relation = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}. So Domain of R = {0, 1, 2, 3, 4, 5,}and Range of R = {5, 6, 7, 8, 9, 10}.

Q12. View Chapter 4 Principle Of Mathematical Induction Exercise 4.1 Q 07. See the solution of this question from **www.theOPGupta.com/** in Class 11 section under NCERT Solutions.

Q13. Let $z = \frac{1+2i}{1-3i}$ $\Rightarrow z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = -\frac{1}{2} + \frac{1}{2}i$

Now let $-\frac{1}{2} = r\cos\theta$ and $\frac{1}{2} = r\sin\theta$ $\Rightarrow \left(-\frac{1}{2}\right)^2 = r^2\cos^2\theta$ and $\left(\frac{1}{2}\right)^2 = r^2\sin^2\theta$

$\frac{1}{4} + \frac{1}{4} = r^2(\cos^2\theta + \sin^2\theta)$ $\therefore r = \frac{1}{\sqrt{2}}$

Also, $\frac{r\sin\theta}{r\cos\theta} = \frac{1/2}{-1/2} \Rightarrow \tan\theta = -1$ $\therefore \theta = \frac{3\pi}{4}$

So polar form of z is $\frac{1}{\sqrt{2}}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$.

Q14. **(i)** $[{}^4C_3\times{}^4C_2]\times 5!$ **(ii)** ${}^6C_2\times{}^4C_3$.

Q15. **(i)** ${}^9C_4\times{}^4C_3$ **(ii)** ${}^9C_4\times{}^4C_3+{}^9C_3\times{}^4C_4$.

Q16. Value of $(a+b)^4-(a-b)^4=8ab(a^2+b^2)$. And $[\sqrt{3}+\sqrt{2}]^4-[\sqrt{3}-\sqrt{2}]^4=40\sqrt{6}$.

Q17. The general term of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^6$, $T_{r+1}=(-1)^r\,{}^6C_r\left(\frac{3}{2}x^2\right)^{6-r}\left(\frac{1}{3x}\right)^r=(-1)^r\,{}^6C_r\left(\frac{3}{2}\right)^{6-r}\left(\frac{1}{3}\right)^r x^{12-3r}$.

For the term independent of x, $12-3r=0$ $\Rightarrow r=4$.

Therefore, $T_5=(-1)^4\,{}^6C_4\left(\frac{3}{2}\right)^{6-4}\left(\frac{1}{3}\right)^4=\frac{5}{12}$.

Q18. LHS : $\cos 52^\circ+\cos 68^\circ+\cos 172^\circ=2\cos\frac{52^\circ+68^\circ}{2}\cos\frac{52^\circ-68^\circ}{2}+\cos(180^\circ-8^\circ)$

$=2\cos 60^\circ\cos(-8^\circ)-\cos 8^\circ=2\times\frac{1}{2}\cos 8^\circ-\cos 8^\circ=0=\text{RHS}.$

Q19. S = V + c, where c is circumstances (constant in the function S). Yes, we agree that Success in life depends on the Values we have acquired. Two life skills are : (i) Punctuality (ii) Truthfulness.

Q20. Let M and P denote the number of students passing in Mathematics and Physics respectively. $\therefore$ n(M) = 80%, n(P) = 72%. Also number of students passing in at least one subjects, $n(M\cup P)$ = 87%. So, $n(M\cap P)$ = 65% = number of students passing in both the subjects.
Let total number of students be x. According to question, 65% of x = 312 $\Rightarrow$ x = 480.

Q21. We have $\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}$ $\Rightarrow \tan\frac{A}{2}\tan\frac{B-C}{2}=\frac{b-c}{b+c}$

RHS : $\frac{b-c}{b+c}=\frac{k\sin B-k\sin C}{k\sin B+k\sin C}$ [By sine rule

$=\frac{k(\sin B-\sin C)}{k(\sin B+\sin C)}=\frac{2\cos\frac{B+C}{2}\sin\frac{B-C}{2}}{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}$ $[\because A+B+C=\pi\Rightarrow\frac{B+C}{2}=\frac{\pi}{2}-\frac{A}{2}$

$=\frac{\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)}{\sin\left(\frac{\pi}{2}-\frac{A}{2}\right)}\tan\frac{B-C}{2}=\frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}\tan\frac{B-C}{2}$

$=\tan\frac{A}{2}\tan\frac{B-C}{2}=\text{LHS}.$

Q22. Let $\sqrt{-4-3i}=x+iy$

On squaring both sides, $-4-3i=x^2-y^2+2xyi$

On equating Re and Im parts on both sides, $x^2-y^2=-4$...(i), $2xy=-3$.

Since $(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2=(-4)^2+(-3)^2=25$

$\Rightarrow x^2+y^2=5$...(ii) $[x^2+y^2\neq-5\ \forall x,y\in R$

Adding (i) and (ii), we get $x=\pm\frac{1}{\sqrt{2}}$,

Subtracting (i) from (ii), we get : $y=\pm\frac{3}{\sqrt{2}}$

So square roots of $-4-3i$ are : $\pm\left(\frac{1}{\sqrt{2}}-\frac{3}{\sqrt{2}}i\right)$.

Q23. Total number of letters in AGAIN = 5. No. of A's = 2.

$\therefore$ the number of words with or without meaning which can be made using all the letters of the word AGAIN $= \frac{5!}{2!} = 60$.

Words starting with A = $4! = 24$, Words starting with G = $\frac{4!}{2!} = 12$, Words starting with I = $\frac{4!}{2!} = 12$

Total words formed so far = 24 + 12 + 12 = 48.

So, 49^{th} word will start from N i.e., 49^{th} word is NAAGI.

Q24. View Chapter 8 Binomial Theorem Miscellaneous Exercise Q 01. See the solution of this question from **www.theOPGupta.com/** in Class 11 section under NCERT Solutions.

Q26. Let $P(n) = \frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number.

For n = 1, $P(1) = \frac{1^5}{5} + \frac{1^3}{3} + \frac{7 \times 1}{15} = \frac{15}{15} = 1 \in N$. $\therefore P(1)$ is true.

Assume that P(k) is true i.e., $P(k) = \frac{k^5}{5} + \frac{k^3}{3} + \frac{7k}{15}$ is a natural number.

Let $\frac{k^5}{5} + \frac{k^3}{3} + \frac{7k}{15} = m$ ...(i)

We have to prove that P(k + 1) is also true whenever P(k) is true i.e.,

$P(k+1) = \frac{(k+1)^5}{5} + \frac{(k+1)^3}{3} + \frac{7(k+1)}{15}$ is a natural number.

Now $\frac{(k+1)^5}{5} + \frac{(k+1)^3}{3} + \frac{7(k+1)}{15} = \frac{k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1}{5} + \frac{k^3 + 3k^2 + 3k + 1}{3} + \frac{7k + 7}{15}$

$= \frac{k^5}{5} + \frac{k^3}{3} + \frac{7k}{15} + \frac{5k^4 + 10k^3 + 10k^2 + 5k + 1}{5} + \frac{3k^2 + 3k + 1}{3} + \frac{7}{15}$

$= m + (k^4 + 2k^3 + 3k^2 + 2k) + \frac{1}{5} + \frac{1}{3} + \frac{7}{15}$ [By using (i)

$= m + k^4 + 2k^3 + 3k^2 + 2k + 1 \in N$ for all $k \in N$

$\therefore$ $P(k+1)$ is also true.

Hence by principle of mathematical induction, P(n) is true for all $n \in N$.