CBSE MATHS SAMPLE PAPER
(Set - 1, 2, 3)

Max. Marks: 100 **Time Allowed: 3 Hours**

SECTION – A

Q01. If a line has direction ratios 2,–1, 2 then, what are its direction cosines?

Q02. Find λ when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4units.

Q03. **(Set I)** Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k},\ \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$, and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

(Set II) Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j}$, $\vec{b} = 2\hat{i} - 3\hat{j}$, and $\vec{c} = 2\hat{i} + 3\hat{k}$.

(Set III) Find the sum of the vectors $\vec{a} = \hat{i} - 3\hat{k}$, $\vec{b} = 2\hat{j} - \hat{k}$, and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$.

Q04. Evaluate: $\int_2^3 \frac{1}{x}dx$.

Q05. Evaluate: $\int (1-x)\sqrt{x}\,dx$.

Q06. **(Set I)** If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23}.

(Set II) If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{32}.

(Set III) If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 5 & 3 & 8 \end{vmatrix}$, write the minor of the element a_{22}.

Q07. If $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$, write the value of x ?

Q08. Simplify: $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$.

Q09. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$.

Q10. Let * be a binary operation on N given by $a*b = \text{LCM}(a, b)$ for all $a, b \in N$. Find 5*7.

SECTION – B

Q11. If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$. **OR** If $\sin y = x\sin(a+y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

Q12. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%?

Q13. **(Set I)** Find the Vector and Cartesian equations of the line passing through the point (1,2,–4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

(Set II) Find the equation of the line passing through the point (–1,3,–2) and perpendicular to the two lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - 3\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$.

(Set III) Find the equation of the line passing through the point P(2,–1,3) and perpendicular to the two lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.

Q14. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a}| = 5, |\vec{b}| = 12$ and $|\vec{c}| = 13$, and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then, find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.

Q15. Solve the differential equation given as: $2x^2 \frac{dy}{dx} - 2xy + y^2 = 0$.

Q16. **(Set I)** Solve the differential equation: $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

(Set II) Find particular solution of the differential equation: $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$; $y = 0$ when $x = 0$.

(Set III) Find particular solution of the differential equation: $xy\frac{dy}{dx} = (x+2)(y+2)$; $y(1) = -1$.

Q17. Evaluate: $\int \sin x \sin 2x \sin 3x \, dx$. **OR** Evaluate: $\int \frac{2}{(1-x)(1+x^2)} dx$.

Q18. Find the point on the curve $y = x^3 - 11x + 5$ at which the equation of tangent is $y = x - 11$.

OR Using differentials, find the approximate value of $\sqrt{49.5}$.

Q19. **(Set I)** If $y = (\tan^{-1} x)^2$, show that $(x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2$.

(Set II) If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

(Set III) If $y = \sin^{-1} x$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

Q20. **(Set I)** Using properties of determinants, prove that: $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$.

(Set II) Using properties of determinants, prove that: $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$.

(Set III) Using properties of determinants, prove that: $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab + bc + ca + abc$.

Q21. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

OR Prove that $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{36}{85}\right)$.

Q22. Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Show that f is one-one and onto and hence find f^{-1}.

SECTION – C

Q23. Find the equation of the plane determined by the points A(3,–1,2), B(5,2,4) and C(–1,–1,6) and hence find the distance between the plane and the point P(6,5,9).

Q24. **(Set I)** Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain 'A' grade and 20% of day scholars attain 'A' grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an 'A' grade, what is the probability that the student is a hostlier?

(Set II) A girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin two times and notes the number of heads obtained. If she obtained exactly two heads, what is the probability that she threw 1, 2, 3 or 4 with the die?

(Set III) Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred balls were both black.

Q25. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of `17.50 per package on nuts and `7 per package of bolts. How many packages of each should be produced each day so as to maximize his profits if he operates his machines for at the most 12 hours a day? Form the above as a linear programming problem and solve it graphically.

Q26. Prove that: $\int_0^{\pi/4}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx=\sqrt{2}\left(\dfrac{\pi}{2}\right)$. **OR** Evaluate $\int_1^3 (2x^2+5x)\,dx$ as a limit of a sum.

Q27. **(Set I)** Using the method of integration, find the area of the region bounded by the lines $3x-2y+1=0$, $2x+3y-21=0$ and $x-5y+9=0$.

(Set II) Using the method of integration, find the area of the region bounded by the lines $3x-y-3=0$, $2x+y-12=0$ and $x-2y-1=0$.

(Set III) Using the method of integration, find the area of the region bounded by the lines $5x-2y-10=0$, $x+y-9=0$ and $2x-5y-4=0$.

Q28. Show that the height of a closed right circular cylinder of given surface and maximum volume, is equal to the diameter of its base.

Q29. Using matrices, solve the following system of linear equations:
$x-y+2z=7$, $3x+4y-5z=-5$, $2x-y+3z=12$.

OR Using elementary operations, find the inverse of the following matrix:

$$\begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}.$$

ANSWERS OF CBSE 2012 DELHI (Set - 1, 2, 3)

Q01. $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$ **Q02.** $\lambda = 5$ **Q03.** **Set I:** $-4\hat{j} - \hat{k}$ **Set II:** $5\hat{i} - 5\hat{j} + 3\hat{k}$

Set III: $3\hat{i} - \hat{j} - 2\hat{k}$ **Q04.** $\log\left(\frac{3}{2}\right)$ **Q05.** $\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + k$

Q06. **Set I:** 7 **Set II:** –11 **Set III:** –7

Q07. 13

Q08. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **Q09.** $\frac{2\pi}{3}$ **Q10.** 35

Q11. $\frac{dy}{dx} = \frac{\log(\cos y) + y\tan x}{\log(\cos x) + x\tan y}$ **Q12.** 3 times

Q13. **Set I:** Cartesian equation: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$. Vector equation: $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

Set II: $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-2}$ **Set III:** $\frac{x-2}{2} = \frac{y+1}{-7} = \frac{z-3}{4}$

Q14. –169 **Q15.** $\log|x| = \frac{2x}{y} + k$

Q16. **Set I:** $y = \tan\left(x + \frac{x^3}{3} + \frac{\pi}{4}\right)$ **Set II:** $(x+1)(2 - e^y) = 1$ **Set III:** $y = x + 2\log|x| + 2\log|y+2| - 2$

Q17. $\frac{\cos 6x}{24} - \frac{\cos 4x}{16} - \frac{\cos 2x}{8} + k$ **OR** $\tan^{-1}x + \frac{1}{2}\log|1 + x^2| - \log|1 - x| + k$

Q18. $(2, -9)$ **OR** 7.036

Q22. $f^{-1} = \frac{3y-2}{y-1}$ **Q23.** $3x - 4y + 3z = 19; \frac{6}{\sqrt{34}}$ units

Q24. **Set I:** $\frac{9}{13}$ **Set II:** $\frac{4}{7}$ **Set III:** $\frac{4}{17}$

Q25. Max.Profit = Rs.73.5 for 3 package of nuts & 3 packages of bolts .

Q26. **OR** $\frac{112}{3}$ **Q27.** **Set I:** $\frac{13}{2}$ sq.units **Set II:** 10 sq.units **Set III:** $\frac{21}{2}$ sq.units

Q29. $x = 2, y = 1, z = 3$ **OR** $\begin{pmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{pmatrix}$.