Series : PTS/15 **Code No. 14/12/15**

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Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 15

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Based on Sample Paper issued by CBSE for Board Exams 2015

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Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION – A

Q01. The position vectors of points A and B are $\vec{a}$ and $\vec{b}$ respectively. P divides AB in the ratio 3 : 1 and Q is mid-point of AP. Find the position vector of Q.

Q02. Find the area of the parallelogram, whose diagonals are $\vec{d}_1 = 5\hat{i}$ and $\vec{d}_2 = 2\hat{j}$.

Q03. If P(2, 3, 4) is the foot of perpendicular from origin to a plane, then write the vector equation of this plane.

Q04. If $\Delta = \begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$, write the cofactor of a_{32} (the element of third row and 2nd column).

Q05. If m and n are the order and degree, respectively of the differential equation $y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$, then write the value of m + n.

Q06. Write the differential equation representing the curve $y^2 = 4ax$, where a is an arbitrary constant.

SECTION – B

Q07. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap-books and pastel sheets made by them using recycled paper, at the rate of Rs. 20, Rs.15 and Rs. 5 per unit respectively. School A sold 25 paper-bags 12 scrap-books and 34 pastel sheets. School B sold 22 paper-bags, 15 scrapbooks and 28 pastel-sheets while school C sold 26 paper-bags, 18 scrap-books and 36 pastel sheets. Using matrices, find the total amount raised by each school.
By such exhibition, which values are inculcated in the students?

Q08. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$, then show that $A^2 - 4A + 7I = O$. Using this result calculate A^3 also.

OR If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix}$, find A^{-1} using elementary row operations.

Q09. If x, y, z are in GP, then using properties of determinants, show that

$$\begin{vmatrix} px+y & x & y \\ py+z & y & z \\ 0 & px+y & py+z \end{vmatrix} = 0, \text{ where } x \neq y \neq z \text{ and p is any real number.}$$

Q10. Evaluate : $\int_{-1}^{1} |x\cos \pi x|\,dx$.

Q11. Evaluate : $\int \frac{1+\sin 2x}{1+\cos 2x} e^{2x} dx$. **OR** Evaluate : $\int \frac{x^4}{(x-1)(x^2+1)} dx$.

Q12. Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 3' given that 'there is at least one head'.

OR How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

Q13. For three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ if $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \times \vec{c} = \vec{b}$, then prove that $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, $|\vec{b}| = |\vec{a}|$ and $|\vec{a}| = 1$.

Q14. Find the equation of the line through the point (1, –1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, –1, 0) and (1, 2, –1), (2, 1, 1).

OR Find the position vector of the foot of perpendicular drawn from the point P(1, 8, 4) to the line joining A(0, –1, 3) and B(5, 4, 4). Also find the length of this perpendicular.

Q15. Solve for x : $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$. **OR** Prove that : $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$.

Q16. If x = sin t, y = sin kt, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2 y = 0$.

Q17. If $y^x + x^y + x^x = a^b$, find $\frac{dy}{dx}$.

Q18. It is given that for the function f (x) = $x^3 + bx^2 + ax + 5$ on [1, 3], Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b.

Q19. Evaluate : $\int \frac{1+3x}{\sqrt{5-2x-x^2}}\,dx$.

SECTION – C

Q20. Let A = {1, 2, 3, …, 9} and R be the relation in $A \times A$ defined by (a, b) R (c, d) if a + d = b + c for a, b, c, d in $A \times A$.

Prove that R is an equivalence relation. Also obtain the equivalence class [(2, 5)].

OR Let f : N ⟶ R be a function defined as f (x) = $4x^2 + 12x + 15$.

Show that f : N ⟶ S is invertible, where S is the range of f. Hence find inverse of f.

Q21. Compute, using integration, the area bounded by the lines x + 2y = 2, y – x = 1 and 2x + y = 7.

Q22. Find the particular solution of the differential equation $xe^{y/x} - y\sin\left(\frac{y}{x}\right) + x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) = 0$, given that $y = 0$, when $x = 1$.

OR Obtain the differential equation of all circles of radius r.

Q23. Show that the lines $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 5\hat{k})$ and $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$ are coplanar. Also, find the equation of the plane containing these lines.

Q24. 40% students of a college reside in hostel and the remaining reside outside. At the end of year, 50% of the hosteliers got A grade while from outside students, only 30% got A grade in the examination. At the end of year, a student of the college was chosen at random and was found to get A grade. What is the probability that the selected student was a hostelier?

Q25. A man rides his motorcycle at the speed of 50km/h. He has to spend Rs. 2 per km on petrol. If he rides it at a faster speed of 80km/h, the petrol cost increases to Rs. 3 per km. He has atmost Rs. 120 to spend on petrol and one hour's time. Using LPP find the maximum distance he can travel.

Q26. A jet of enemy is flying along the curve y = x^2 + 2 and a soldier is placed at the point (3, 2). Find the minimum distance between the soldier and the jet.

SECTION – A

Q01. $\dfrac{5\vec{a}+3\vec{b}}{8}$ **Q02.** Area of the parallelogram $=\dfrac{1}{2}\left|\vec{d}_1\times\vec{d}_2\right|=5$ Sq.units

Q03. $\vec{r}.(2\hat{i}+3\hat{j}+4\hat{k})=29$ **Q04**. -14 **Q05**. $m+n=4$ **Q06**. $2x\dfrac{dy}{dx}-y=0$ $[6\times1=6$

SECTION – B

Q07. Sale matrix for A, B and C is $\begin{pmatrix}25 & 12 & 34\\ 22 & 15 & 28\\ 26 & 18 & 36\end{pmatrix}$ ½

Price matrix is $\begin{pmatrix}20\\ 15\\ 5\end{pmatrix}$ ½

$\therefore \begin{pmatrix}25 & 12 & 34\\ 22 & 15 & 28\\ 26 & 18 & 36\end{pmatrix}\begin{pmatrix}20\\ 15\\ 5\end{pmatrix}=\begin{pmatrix}500+180+170\\ 440+225+140\\ 520+270+180\end{pmatrix}=\begin{pmatrix}850\\ 805\\ 970\end{pmatrix}$ ½

So amount raised by A is Rs.850, by B is Rs.805 and by C is Rs.970. ½

Values : • Helping the orphans • Use of recycled paper 1 + 1

Q08. $A^2=A.A=\begin{pmatrix}2 & 3\\ -1 & 2\end{pmatrix}\begin{pmatrix}2 & 3\\ -1 & 2\end{pmatrix}=\begin{pmatrix}1 & 12\\ -4 & 1\end{pmatrix}$ 1

$\therefore A^2-4A+7I=\begin{pmatrix}1 & 12\\ -4 & 1\end{pmatrix}-4\begin{pmatrix}2 & 3\\ -1 & 2\end{pmatrix}+7\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}=O$ 2

Now $A^2-4A+7I=O\Rightarrow A^2=4A-7I\Rightarrow A^3=A.A^2=4A^2-7A=4(4A-7I)-7A=9A-28I$

$\therefore A^3=9A-28I=\begin{pmatrix}18 & 27\\ -9 & 18\end{pmatrix}-\begin{pmatrix}28 & 0\\ 0 & 28\end{pmatrix}=\begin{pmatrix}-10 & 27\\ -9 & -10\end{pmatrix}$ 1

OR $\because A=IA$ $\Rightarrow\begin{pmatrix}1 & -1 & 0\\ 2 & 5 & 3\\ 0 & 2 & 1\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}A$ ½

By $R_2\to R_2-2R_1$, $\begin{pmatrix}1 & -1 & 0\\ 0 & 7 & 3\\ 0 & 2 & 1\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ -2 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}A$ 1

By $R_2\to R_2-3R_3$, $\begin{pmatrix}1 & -1 & 0\\ 0 & 1 & 0\\ 0 & 2 & 1\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ -2 & 1 & -3\\ 0 & 0 & 1\end{pmatrix}A$ 1

By $R_1\to R_1+R_2$ & $R_3\to R_3-2R_2$, $\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}-1 & 1 & -3\\ -2 & 1 & -3\\ 4 & -2 & 7\end{pmatrix}A$ 1

$\because A^{-1}A=I,\quad \therefore A^{-1}=\begin{pmatrix}-1 & 1 & -3\\ -2 & 1 & -3\\ 4 & -2 & 7\end{pmatrix}$ ½

Q09. Let $\Delta = \begin{vmatrix} px+y & x & y \\ py+z & y & z \\ 0 & px+y & py+z \end{vmatrix}$

By $C_1 \to C_1 - pC_1 - C_3$, $\Delta = \begin{vmatrix} 0 & x & y \\ 0 & y & z \\ -p^2x - py - py - z & px+y & py+z \end{vmatrix}$ 1 ½

Expanding along C_1, $\Delta = (-p^2x - 2py - z)(xz - y^2)$ 1

$\because$ x,y,z are in GP, so $xz = y^2 \Rightarrow xz - y^2 = 0$ $\quad \therefore \Delta = 0$ 1 ½

Q10. $\int_{-1}^{1} |x\cos\pi x|\,dx = 2\int_{0}^{1} |x\cos\pi x|\,dx$ 1

$\Rightarrow \quad = 2\int_{0}^{1/2} x\cos\pi x\,dx - 2\int_{1/2}^{1} x\cos\pi x\,dx$ 1

$\Rightarrow \quad = 2\left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_0^{1/2} - 2\left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{1/2}^{1}$ 1

$\Rightarrow \quad = 2\left[\frac{1}{2\pi} - \frac{1}{\pi^2}\right] - 2\left[-\frac{1}{\pi^2} - \frac{1}{2\pi}\right] = \frac{2}{\pi}.$ 1

Q11. Put $2x = t \Rightarrow dx = \frac{1}{2}dt$ $\quad \therefore \int \frac{1+\sin 2x}{1+\cos 2x} e^{2x} dx = \frac{1}{2}\int \frac{1+\sin t}{1+\cos t} e^{t} dt$ ½

$\Rightarrow \quad = \frac{1}{2}\int \left(\frac{1}{2\cos^2(t/2)} + \frac{2\sin(t/2)\cos(t/2)}{2\cos^2(t/2)}\right) e^{t} dt$ 1

$\Rightarrow \quad = \frac{1}{2}\int \left(\frac{\sec^2(t/2)}{2} + \tan(t/2)\right) e^{t} dt$ 1

$\because f(t) = \tan(t/2), f'(t) = \frac{\sec^2(t/2)}{2}$ $\quad \therefore$ using $\int [f(t) + f'(t)] e^{t} dt = f(t)e^{t} + C$ ½

$\therefore \int \frac{1+\sin 2x}{1+\cos 2x} e^{2x} dx = \frac{1}{2} e^{t} \tan\left(\frac{t}{2}\right) + C = \frac{1}{2} e^{2x} \tan x + C.$ 1

OR $\int \frac{x^4}{(x-1)(x^2+1)} dx = \int \left(x + 1 + \frac{1}{(x-1)(x^2+1)}\right) dx \text{ ...(i)}$ 1

Consider $\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} \Rightarrow A = \frac{1}{2}, B = C = -\frac{1}{2}$

By (i), $\int \frac{x^4}{(x-1)(x^2+1)} dx = \int \left(x + 1 + \frac{1}{2(x-1)} - \frac{x}{2(x^2+1)} - \frac{1}{2(x^2+1)}\right) dx$ 1

$\Rightarrow \quad = \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + C.$ 1 + 1

Q12. Let E : Die shows a number greater than 3 and F : there is at least one head.
$\Rightarrow$ E : {H4, H5, H6}, F : {HT, H1, H2, H3, H4, H5, H6} ½ + ½
$\therefore$ P(F) = 1 – ¼ = ¾, P(E∩F) = 3/12 = ¼ 1 + 1

$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/4}{3/4} = \frac{1}{3}$ 1

OR We have $p = \frac{1}{2}, q = \frac{1}{2}$; let the coin be tossed n times.

$P(r \ge 1) > \frac{90}{100}$ or $1 - P(r < 1) > \frac{90}{100}$ ½ + ½

$\Rightarrow 1-\frac{90}{100} > P(r=0)$ ½

$\Rightarrow \frac{1}{10} > {}^nC_0\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^0$ $\qquad \Rightarrow \frac{1}{2^n} < \frac{1}{10}$ 1 ½

$\Rightarrow 2^n > 10$ $\qquad \therefore n = 4$ 1

Q13. We are given that

$\left.\begin{array}{l}\vec{a}\times\vec{b}=\vec{c}\Rightarrow\vec{a}\perp\vec{c}\text{ and }\vec{b}\perp\vec{c}\\ \vec{a}\times\vec{c}=\vec{b}\Rightarrow\vec{a}\perp\vec{b}\text{ and }\vec{c}\perp\vec{b}\end{array}\right\}\Rightarrow\vec{a}\perp\vec{b}\perp\vec{c}\ \ldots\text{(i)}$ 1

Now, $\vec{a}\times\vec{b}=\vec{c}\Rightarrow|\vec{a}\times\vec{b}|=|\vec{c}|\Rightarrow|\vec{a}||\vec{b}|\sin\frac{\pi}{2}=|\vec{c}|\Rightarrow|\vec{a}||\vec{b}|=|\vec{c}|\ \ldots\text{(ii)}$ [by (i), $\vec{a}\perp\vec{b}$ 1

And, $\vec{a}\times\vec{c}=\vec{b}\Rightarrow|\vec{a}||\vec{c}|\sin\frac{\pi}{2}=|\vec{b}|\Rightarrow|\vec{a}||\vec{c}|=|\vec{b}|\ \ldots\text{(iii)}$ [by (i), $\vec{a}\perp\vec{c}$ 1

By (ii) ÷ (iii), we get : $|\vec{c}|^2=|\vec{b}|^2\Rightarrow|\vec{c}|=|\vec{b}|$.

Substitute $|\vec{c}|=|\vec{b}|$ in (ii) to obtain, $|\vec{a}|=1$. 1

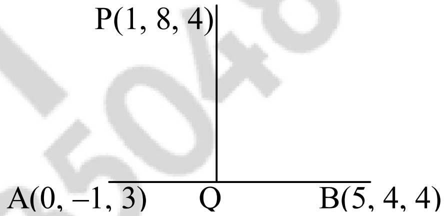
Q14. The d.r.'s of line L_1 joining (4, 3, 2) and (1, –1, 0) are 3, 4, 2 ½

The d.r.'s of line L_2 joining (1, 2, –1) and (2, 1, 1) are 1, –1, 2. ½

A vector perpendicular to both the lines is $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 4 & 2\\ 1 & -1 & 2\end{vmatrix}=10\hat{i}-4\hat{j}-7\hat{k}$ 1 ½

$\therefore$ eq. of the line through (1, –1, 1) and $\perp$ to L_1 and L_2 is : $\vec{r}=(\hat{i}-\hat{j}+\hat{k})+\lambda(10\hat{i}-4\hat{j}-7\hat{k})$ 1½

OR Equation of line AB is $\vec{r}=(-\hat{j}+3\hat{k})+\lambda(5\hat{i}+5\hat{j}+\hat{k})$ 1



$\therefore$ Point Q is $(5\lambda, -1+5\lambda, 3+\lambda)$ ½

$\overline{PQ}=(5\lambda-1)\hat{i}+(5\lambda-9)\hat{j}+(\lambda-1)\hat{k}$ ½

$\because PQ\perp AB\Rightarrow 5(5\lambda-1)+5(5\lambda-9)+1(\lambda-1)=0\Rightarrow\lambda=1$ ½

$\therefore$ foot of perpendicular is Q(5, 4, 4) ½

length of perpendicular is $PQ=\sqrt{4^2+(-4)^2+0^2}=4\sqrt{2}$ units . 1

Q15. $\sin^{-1}6x+\sin^{-1}6\sqrt{3}x=-\frac{\pi}{2}$ $\qquad \Rightarrow \sin^{-1}6x=-\frac{\pi}{2}-\sin^{-1}6\sqrt{3}x$ ½

$\Rightarrow \sin\sin^{-1}6x=\sin\left(-\frac{\pi}{2}-\sin^{-1}6\sqrt{3}x\right)$ $\qquad \Rightarrow 6x=-\sin\left(\frac{\pi}{2}+\sin^{-1}6\sqrt{3}x\right)$ ½

$\Rightarrow 6x=-\cos\left(\sin^{-1}6\sqrt{3}x\right)=-\sqrt{1-108x^2}$ 1

$\Rightarrow 36x^2=1-108x^2$ $\qquad \Rightarrow 144x^2=1$ $\qquad \Rightarrow x=\pm\frac{1}{12}$ 1

Since $x=\frac{1}{12}$ does not satisfy the given equation, $\therefore x=-\frac{1}{12}$. 1

OR LHS : $2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=2\tan^{-1}\frac{3}{4}-\tan^{-1}\frac{17}{31}$ 1

$\Rightarrow \qquad =\tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right)-\tan^{-1}\frac{17}{31}$ 1

$\Rightarrow \qquad =\tan^{-1}\frac{24}{7}-\tan^{-1}\frac{17}{31}$ 1

$$\Rightarrow \quad = \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\times\frac{17}{31}}\right) = \tan^{-1}(1) = \frac{\pi}{4} = \text{RHS.}$$ 1

Q16. Here x = sin t, y = sin kt

$\therefore \frac{dx}{dt} = \cos t, \frac{dy}{dt} = k\cos kt \qquad \Rightarrow \frac{dy}{dx} = k\frac{\cos kt}{\cos t}$ 1

$\Rightarrow \cos t\frac{dy}{dx} = k\cos kt \qquad \Rightarrow \cos^2 t\left(\frac{dy}{dx}\right)^2 = k^2\cos^2 kt$ ½

$\Rightarrow (1-\sin^2 t)\left(\frac{dy}{dx}\right)^2 = k^2(1-\sin^2 kt) \quad \Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = k^2(1-y^2)$ 1

Differentiating w.r.t. x both the sides,

$(1-x^2)\times 2\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2(-2x) = -2k^2y\frac{dy}{dx}$ 1

$\therefore \ (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2y = 0$ ½

Q17. Let $u = y^x, v = x^y, w = x^x$

(i) $u = y^x \ \Rightarrow \log u = x\log y \qquad \Rightarrow \frac{du}{dx} = y^x\left[\log y + \frac{x}{y}\frac{dy}{dx}\right]$ 1

(ii) $v = x^y \ \Rightarrow \log v = y\log x \qquad \Rightarrow \frac{dv}{dx} = x^y\left[\log x\frac{dy}{dx} + \frac{y}{x}\right]$ ½

(iii) $w = x^x \Rightarrow \log w = x\log x \qquad \Rightarrow \frac{dw}{dx} = x^x[\log x + 1]$ ½

Now $y^x + x^y + x^x = a^b \qquad \Rightarrow u + v + w = a^b \qquad \Rightarrow \frac{du}{dx}+\frac{dv}{dx}+\frac{dw}{dx} = 0$

$\Rightarrow y^x\left[\log y + \frac{x}{y}\frac{dy}{dx}\right] + x^y\left[\log x\frac{dy}{dx}+\frac{y}{x}\right] + x^x[\log x + 1] = 0$ 1

$\therefore \ \frac{dy}{dx} = -\frac{y^x\log y + yx^{y-1} + x^x[\log x + 1] = 0}{\log x + x\,y^{x-1}}$ 1

Q18. Given f (x) = $x^3 + bx^2 + ax + 5$ on [1, 3]

$\Rightarrow f'(x) = 3x^2 + 2bx + a \Rightarrow f'(c) = 3c^2 + 2bc + a = 0 \ \Rightarrow 3\left(2+\frac{1}{\sqrt{3}}\right)^2 + 2b\left(2+\frac{1}{\sqrt{3}}\right) + a = 0 \ldots$(i) 1+1

Also $f(1) = f(3) \Rightarrow b + a + 6 = 32 + 9b + 3a$ or, $a + 4b = -13 \ldots$(ii) 1

Solving (i) and (ii), we get : $a = 11, b = -6$. 1

Q19. Let $1+3x = A\frac{d}{dx}[5-2x-x^2] + B \Rightarrow 1+3x = A(-2x-2)+B \Rightarrow A = -\frac{3}{2}, B = -2$ 1

$\int\frac{1+3x}{\sqrt{5-2x-x^2}}dx = -\frac{3}{2}\int\frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int\frac{1}{\sqrt{5-2x-x^2}}dx$ 1

$\Rightarrow \quad = -\frac{3}{2}\left[2\sqrt{5-2x-x^2}\right] - 2\int\frac{dx}{\sqrt{[\sqrt{6}]^2-[x+1]^2}}$ 1

$\Rightarrow \quad = -3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + C.$ 1

SECTION – C

Q20. Reflexivity : Let (a, b) be an arbitrary element of $A\times A$. Then,

(a, b)$\in A\times A \Rightarrow$ a, b$\in$A.

So, a + b = b + a $\Rightarrow$ (a, b) R (a, b)

Thus, (a, b) R (a, b) $\forall$ (a, b)$\in A\times A$. Hence R is reflexive. 1
Symmetry : Let a, b, c, d$\in A\times A$ be such that (a, b) R (c, d).
Then, $a + d = b + c \Rightarrow c + b = d + a \Rightarrow$ (c, d) R (a, b).
Thus, (a, b) R (c, d) $\Rightarrow$ (c, d) R (a, b) $\forall$ a, b, c, d$\in A\times A$. Hence R is symmetric. 1
Transitivity : Let a, b, c, d, e, f $\in A\times A$ be such that (a, b) R (c, d) and (c, d) R (e, f).
Then, $a + d = b + c$ and $c + f = d + e \Rightarrow (a + d) + (c + f) = (b + c) + (d + e) \Rightarrow a + f = b + e$
$\Rightarrow$ (a, b) R (e, f). That is, (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow$ (a, b) R (e, f) $\forall$ a, b, c, d, e, f$\in A\times A$.
Hence R is transitive. 2
Since R is reflexive, symmetric and transitive so, R is an equivalence relation as well. ½
For the equivalence class of [(2, 5)], we need to find (a, b) s. t. (a, b) R (2, 5) $\Rightarrow a + 5 = b + 2$
$\Rightarrow b - a = 3$. So, [(2, 5)] = {(1,4), (2,5), (3,6), (4,7), (5,8), (6,9)}. 1 ½

OR Let y be an arbitrary element of range of function. Then $y = 4x^2 + 12x + 15$, for some x in N, which implies that $y = (2x + 3)^2 + 6$. This gives $x = \dfrac{\sqrt{y-6}-3}{2}$, as $y \ge 6$. 1

Let us define $g : S \to N$ by $g(y) = \dfrac{\sqrt{y-6}-3}{2}$. 1

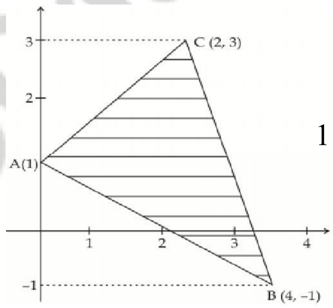
Now, $gof\,(x) = g\left(f\,(x)\right) = g(4x^2+12x+15) = g\left((2x+3)^2+6\right) = \dfrac{\sqrt{\left((2x+3)^2+6\right)-6}-3}{2} = x$. 1

And, $fog\,(y) = f\left(g\,(y)\right) = f\left(\dfrac{\sqrt{y-6}-3}{2}\right) = \left(2\left(\dfrac{\sqrt{y-6}-3}{2}\right)+3\right)^2 + 6 = y$. 1

Hence, gof = I_N and fog = I_S. This implies that f is invertible with $f^{-1} = g$. 1

So, $f^{-1}(x) = \dfrac{\sqrt{x-6}-3}{2}$. 1

Q21. Let the lines be, AB : $x + 2y = 2$, CA : $y - x = 1$ and BC: $2x + y = 7$.
$\therefore$ Points of intersection are A(0, 1), B(4, –1) and C(2, 3) 1



1 ½

Required area $= \dfrac{1}{2}\int_{-1}^{3}(7-y)dy - \int_{-1}^{1}(2-2y)dy - \int_{1}^{3}(y-1)dy$ 1 ½

$\Rightarrow \quad = \dfrac{1}{2}\left[7y - \dfrac{1}{2}y^2\right]_{-1}^{3} - \left[2y - y^2\right]_{-1}^{1} - \left[\dfrac{1}{2}y^2 - y\right]_{1}^{3}$ 1 ½

$\Rightarrow \quad = 12 - 4 - 2 = 6$ Sq.units ½

Q22. Given differential equation is homogeneous.

$\therefore y = vx \qquad \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ ½

So, $\dfrac{dy}{dx} = \dfrac{y\sin\left(\dfrac{y}{x}\right) - xe^{y/x}}{x\sin\left(\dfrac{y}{x}\right)} \qquad \Rightarrow v + x\dfrac{dv}{dx} = \dfrac{vx\sin\left(\dfrac{vx}{x}\right) - xe^{vx/x}}{x\sin\left(\dfrac{vx}{x}\right)} = \dfrac{v\sin v - e^v}{\sin v}$ 1

$\Rightarrow v + x\dfrac{dv}{dx} = v - \dfrac{e^v}{\sin v}$ or $x\dfrac{dv}{dx} = -\dfrac{e^v}{\sin v}$

$\therefore \int e^{-v}\sin v\,dv = -\int\dfrac{dx}{x} \qquad \Rightarrow I_1 = -\log x + C_1 ...(i)$ 1

Now $\therefore I_1 = \int e^{-v}\sin v\,dv = \sin v\int e^{-v}dv + \int e^{-v}\cos v\,dv$

$\Rightarrow \quad = -e^{-v}\sin v - e^{-v}\cos v - \int e^{-v}\sin v\,dv \qquad \Rightarrow I_1 = -\dfrac{1}{2}e^{-v}(\sin v + \cos v) + C_2$ 1

Putting value of I_1 in (i), $-\dfrac{1}{2}e^{-v}(\sin v + \cos v) = -\log x + C_1 + C_2$

$e^{-y/x}\left(\sin\frac{y}{x}+\cos\frac{y}{x}\right)=\log x^2+C$, where $C=-2C_1-2C_2$ 1

As it is given that $y=0$, when $x=1$, so C = 1. 1

Hence the solution is $e^{-y/x}\left(\sin\frac{y}{x}+\cos\frac{y}{x}\right)=\log x^2+1$ ½

OR Let the equation of circle be $(x-a)^2+(y-b)^2=r^2 \ldots(i)$

$\Rightarrow 2(x-a)+2(y-b)\frac{dy}{dx}=0 \ldots(ii)$ ½

$\Rightarrow 1+(y-b)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0 \ldots(iii)$ ½

$\therefore (y-b)=-\frac{1+(y_1)^2}{y_2}$ 1 ½

From (ii), $(x-a)=\left[\frac{1+(y_1)^2}{y_2}\right]y_1$ 1 ½

Putting these values in (i), we get : $\left[\frac{1+(y_1)^2}{y_2}\right]^2(y_1)^2+\left[-\frac{1+(y_1)^2}{y_2}\right]^2=r^2$ 1

or $\left[1+(y_1)^2\right]^3=(r\,y_2)^2$. 1

Q23. Here $\vec{a}_1=-3\hat{i}+\hat{j}+5\hat{k}, \vec{b}_1=-3\hat{i}+\hat{j}+5\hat{k}$, $\vec{a}_2=-\hat{i}+2\hat{j}+5\hat{k}, \vec{b}_2=-\hat{i}+2\hat{j}+5\hat{k}$ ½

Now $(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)=\begin{vmatrix}2&1&0\\-3&1&5\\-1&2&5\end{vmatrix}=2(-5)-1(-15+5)=-10+10=0$ 1 ½

$\therefore$ Given lines are coplanar. ½

Perpendicular vector to the plane $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-3&1&5\\-1&2&5\end{vmatrix}=-5\hat{i}+10\hat{j}-5\hat{k}$ or $\hat{i}-2\hat{j}+\hat{k}$ 2

$\therefore$ Eq. of plane : $\vec{r}.(\hat{i}-2\hat{j}+\hat{k})=(\hat{i}-2\hat{j}+\hat{k}).(-3\hat{i}+\hat{j}+5\hat{k})$ $\Rightarrow x-2y+z=0$ 1 ½

Q24. Let E_1: Student resides in the hostel, E_2: Student resides outside the hostel, A: Getting A grade in the examination.

$P(E_1)=\frac{40}{100}=\frac{2}{5}, P(E_2)=\frac{3}{5}, P(A|E_1)=\frac{50}{100}=\frac{1}{2}, P(A|E_2)=\frac{30}{100}=\frac{3}{10}$. ½ + ½ + 1 + 1

By Bayes' theorem, $P(E_1|A)=\frac{P(A|E_1)P(E_1)}{P(A|E_1)P(E_1)+P(A|E_2)P(E_2)}$ 1

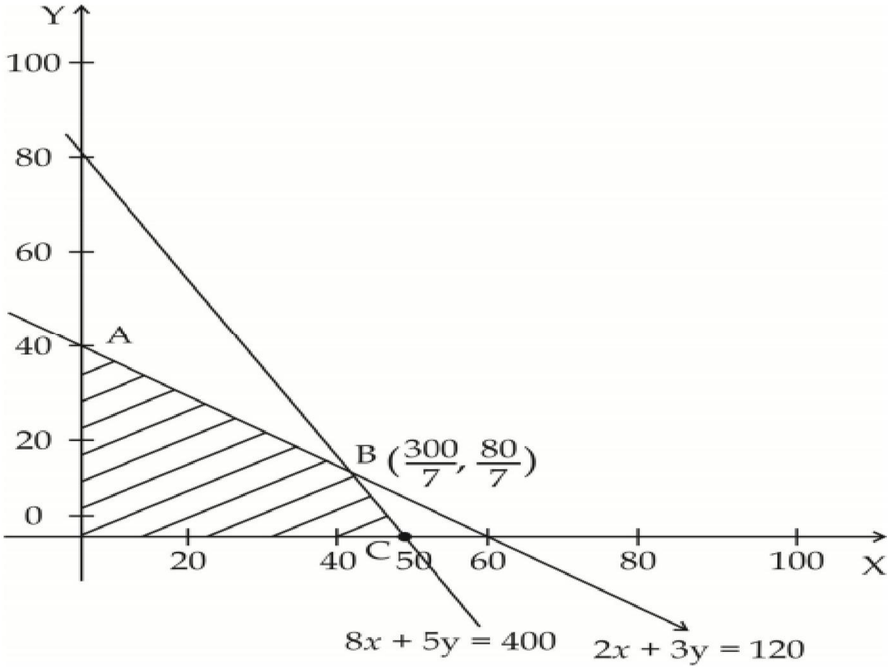
$\Rightarrow \quad =\frac{\frac{2}{5}\times\frac{1}{2}}{\frac{2}{5}\times\frac{1}{2}+\frac{3}{5}\times\frac{3}{10}}=\frac{10}{19}$ 1 + 1

Q25. Let the distance travelled @ 50 km/h be x km and that @ 80 km/h be y km.
To Maximize : D = x + y

Subject to constraints : 2x + 3y ≤ 120, $\frac{x}{50}+\frac{y}{80}\le 1$ or $8x+5y\le 400, x\ge 0, y\ge 0$ ½ + 2

Vertices are (0, 40), (300/7, 80/7), (50, 0)
Maximum D is at (300/7, 80/7)

Maximum D = $\frac{380}{7}$ km $=54\frac{2}{7}$ km. 1 ½



2

Q26. Let P(x, y) be the position of the jet and the soldier is placed at A(3, 2).

$\Rightarrow AP = \sqrt{(x-3)^2 + (y-2)^2}$ …(i) ½

As $y = x^2 + 2 \Rightarrow x^2 = y - 2$ …(ii) ½

$\Rightarrow AP^2 = (x-3)^2 + x^4 = z$ (Say)

$\Rightarrow \frac{dz}{dx} = 2(x-3) + 4x^3$ and $\frac{d^2z}{dx^2} = 2 + 12x^2$ 2

For local points of maxima/minima, $\frac{dz}{dx} = 0 \Rightarrow 2(x-3) + 4x^3 = 0 \Rightarrow x = 1$ 1

And $\frac{d^2z}{dx^2}(\text{at } x = 1) = 14 > 0$ 1

$\therefore$ z is minimum when $x = 1$, $y = 1 + 2 = 3$

Also minimum distance $= \sqrt{(3-1)^2 + 1^2} = \sqrt{5}$ units . 1