

Mathematics XII

Total Marks -100 Time- 3 h

Section – A, One marks each, Section – B, four marks Each, Section – C, Six marks each

SECTION-A

1. If $\sin^{-1}x + \cos^{-1}\frac{2}{3} = \frac{\pi}{2}$ the value of x is=?
2. If $a*b = a+b-ab$ the find the value of $3*4$
3. Find the equation of the plane which makes equal intercepts on the coordinate axis and pass through the point $(-2,3,0)$
4. If A is a square matrix of order 3 such that $|A|= 7$ then find $|3A|$
5. Find the degree and order of the differential equation $(\frac{d^2y}{dx^2})^3 + (\frac{dy}{dx})^4 + 2y=0$
6. Evaluate $\int \frac{d(\log x)}{\log x}$

SECTION-B

7. If $(a + bx)e^{y/x} = x$ then prove that $x^3 \frac{d^2y}{dx^2} = (x \frac{dy}{dx} - y)^2$
8. Verify the LMVT for the function $f(x) = x^2 + 2x + 3$ in $[4,6]$
9. Solve $\tan^{-1}x + \tan^{-1}2x = \frac{\pi}{4}$
10. Express the matrix as a sum of a symmetric and a skew-symmetric matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$
11. if $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that $f(f(x))=x$. also find the inverse of $f(x)$
12. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ then show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$
13. Discuss the continuity of the function $F(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ at $x=0$
14. Show that $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (b+a)^2 \end{vmatrix} = 2abc(a+b+c)$
15. Solve $(1+x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$ given that $y=0, x=1$

16. Evaluate $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$

17. Evaluate $\int_0^\pi \frac{x \sin x}{1 + \sin x} dx$

18. A ladder 13 m is leaning against a vertical wall the bottom of the ladder is dragged away from the wall along the ground at the speed of 2 cm/sec. how fast is its height is decreasing when the foot of the ladder is 5 m away from the wall. **OR**

Find the equation of the plane through the line of intersection of the planes $2x+y-z=3$ & $5x-3y+4z+9$ and parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$

19. Prove that $|\vec{a} \times \vec{b}|^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$ **OR**

Four defective bulbs mixed with 10 good ones three bulbs taken at random at random, find the probability distribution of the defective bulbs

SECTION-C

20. Using limit as sum evaluate $\int_1^3 (2x^2 + x + 9) dx$ **OR**

Find the area of the region bounded by the curves $x^2+y^2=4$ & $y = |x|$.

21. Two companies decide to award there employee for the three values of resourcefulness, competitance and determination in the form of prizes at the rate of Rs x,y,z respectively per person. The first company decide to awarded respectively 4,3,2 employees with a total prize money Rs 37000. The other company decide to awarded respectively 5,3,4 employees with a total prize money of Rs 47000. If all the three prizes per person together amount of Rs 12000 then using the matrix method find the values of x,y,z .

22. Show that the greatest cone that can be inscribe in a given sphere of radius R is $\frac{8}{27}$ part of the volume of the sphere.

23. Evaluate $\int_0^4 \{|x-1| + |x-2| + |x-4|\} dx$ **OR** $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$

24. A manufacturer of patent medicines is preparing a production plan on medicines A & B. there are sufficient raw materials available to make 20,000 bottles of A and 40,000 bottles of B, but there are only 45,000 bottles into which either of the medicines can be put. Further , it takes 3 hours to prepare enough material to fill 1000 bottles of A, it takes 1 hour to prepare enough material to fill 1000 bottles

of B and there are 66 hours available for this operation. The profit is Rs 8 per bottles of A and Rs 7 per bottles of B. how should the manufacturer schedule his production in order to maximize his profit.

25. There are three probable candidates A, B, C for the post of a Principal. The chances of their selections are in the proportions of 4:2:3 respectively. The probability that A ,if selected, will introduce co-education in collage is 0.3. the probability of B and C doing the same are respectively 0.5 and 0.8. what is the probability that there will be co-education in the college? Find the probability that principal B introduced co-education in college.

26. A point on the hypotenuse of a right angle triangle is a & b distance apart from the two legs of right angle then show that the minimum length of the hypotenuse is $(a^2+b^2)^{3/2}$

27. $f(x)= 2x^3-9x^2+12x+15$; find the interval where f(x) is increasing & decreasing.

Best of Luck

N.D. DEBNATH