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Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII - 20

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Time Allowed : 180 Minutes

Max. Marks : 100

SECTION - A

- Q01. Find the differential equation of all non-vertical lines in a plane.
- Q02. If $\sin^{-1} : [-1, 1] \rightarrow \left[\frac{\pi}{2}, \frac{3\pi}{2} \right]$ is a function, then write the value of $\sin^{-1} \left(-\frac{1}{2} \right)$.
- Q03. Write the integrating factor for the linear differential equation : $x \frac{dy}{dx} - y = x^2$.
- Q04. What is the distance between the planes $3x + 4y - 7 = 0$ and $6x + 8y + 6 = 0$?
- Q05. If A and B are square matrices of order 3×3 such that $|A| = -1$ and $|B| = 4$, then what is the value of $|3(AB)|$.
- Q06. What is the value of $|\hat{i} - \hat{j}|^2$?

SECTION - B

- Q07. If $y = \log \left(x^{\sin x} + \frac{x^2 - 1}{x^2 + 1} \right)$, find $\frac{dy}{dx}$.
- Q08. Show that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2 + b^2 + c^2)$ using properties of determinants.
- Q09. Evaluate : $\int_0^{\pi/6} \sin^4 x \cos^3 x \, dx$. (OR) Evaluate : $\int_{-\pi/4}^{\pi/4} \log(\sin x + \cos x) \, dx$.
- Q10. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹80, ₹60 and ₹40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra. What are the benefits of reading books over getting information from internet?
- Q11. Given $f(x) = |x-1| + |x+2|$. Show that f is continuous at $x=1$ but fails to be differentiable at $x=-2$.
- (OR) For what value of k, is $f(x) = \begin{cases} \frac{\sqrt{5x+2} - \sqrt{4x+4}}{x-2}, & \text{if } x \neq 2 \\ k, & \text{if } x = 2 \end{cases}$ continuous at $x = 2$?
- Q12. Find the equation of the plane passing through the point A(1, 2, 1) and perpendicular to the line joining the points P(1, 4, 2) and Q(2, 3, 5). Also, find the distance of this plane from the line $\frac{x+2}{2} = \frac{y-5}{-1} = \frac{z-7}{-1}$.
- Q13. Write $\tan^{-1} \left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right)$ in the simplest form.
- (OR) $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$.

Q14. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$. Verify that $A (\text{adj. } A) = |A| I$.

Q15. Evaluate : $\int \frac{dx}{\cos x + \sin 2x}$.

Q16. Find the equation of tangent to the curve $x = \sin 3t, y = \cos 2t$ at $t = \frac{\pi}{4}$.

(OR) Find the intervals on which the function given as $f(x) = \sin^4 x + \cos^4 x$ in $\left[0, \frac{\pi}{2}\right]$ is

(a) increasing (b) decreasing.

Q17. Show that the four points whose position vectors are $6\hat{i} - 7\hat{j}$, $16\hat{i} - 19\hat{j} - 4\hat{k}$, $3\hat{j} - 6\hat{k}$ and $2\hat{i} - 5\hat{j} + 10\hat{k}$ are coplanar.

Q18. Evaluate : $\int x (\log x)^2 dx$.

Q19. A director of selection committee is biased so that he selects his relatives for a job two times as likely as others. If there are two posts for a job, find the probability distribution for selection of his relatives.

SECTION - C

Q20. A manufacturer produces two models of bikes : model X and model Y. Model X takes a 6 man-hours to make per unit, while model Y takes 10 man-hours per unit. There is a total of 450 man-hours available per week. Handling and marketing costs are ₹2000 and ₹1000 per unit for models X and Y respectively. The total funds available for these purposes are ₹80,000 per week. Profits per unit for models X and Y are ₹1000 and ₹500 respectively. How many bikes of each model should the manufacturer produce so as to yield a maximum profit? Find the maximum profit also.

Q21. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = 2x - 4$ and $g(x) = \frac{1}{2x + 6}$. Find a new function $\frac{f}{g}(x)$. Prove that the newly formed function is bijective. Also find its inverse.

Q22. A wire of length 34 cm is to be cut into two pieces. One of the pieces will be bent into shape of a square and the other into shape of a rectangle such that the length of rectangle is twice its breadth. Where the wire should be cut so that the sum of the areas of the square and rectangle is minimum? Also, find the minimum area.

Q23. Using integrals, find the area of ΔABC whose vertices have the coordinates as $A(-1, 1)$, $B(0, 5)$ and $C(3, 2)$. **(OR)** Find the area of the region $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.

Q24. Solve : $x \frac{dy}{dx} \sin\left(\frac{y}{x}\right) + x = y \sin\left(\frac{y}{x}\right), y(1) = \frac{\pi}{2}$.

Q25. There are three coins. One is a biased coin that comes up with tail 60% of the times, the second is also a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it showed heads. What is the probability that it was the unbiased coin?

Q26. Find the distance of point $P(3, 4, 5)$ from the plane $x + y + z = 2$ measured parallel to the line whose equation is given as $2x = y = z$.

(OR) Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also, find the equation of the plane containing these two lines.