Series : PTS/13 **Code No. 14/11/13**

Roll No. | | | | | | | |

Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 13

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Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION – A

Q01. In the matrix equation $\begin{bmatrix} 11 & 16 \\ 7 & 10 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, apply $C_2 \to C_2 - C_1$ on both sides.

Q02. Let f be a function defined as $f(x) = \dfrac{1}{2 - \sin 3x}$. Write the range of $f(x)$.

Q03. For a non-singular matrix A, find $|adj(A^T)|$ if $A^{-1} = \begin{bmatrix} 1/5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Q04. Write the principal value of $\sin^{-1}(\sin 10)$.

Q05. Evaluate the integral : $\int_{-1}^{2} e^{-\log_e x} dx$.

Q06. Check if the points A(4, 2, 3), B(1, 3, 1) and C(–5, 5,–2) are collinear points? Use vectors.

SECTION – B

Q07. a) Check if $\vec{a} = -4\hat{i} - 6\hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{c} = -8\hat{i} - \hat{j} + 3\hat{k}$ are coplanar vectors.

b) Let $f : [1,\infty) \to [1,\infty)$ be defined as $f(x) = 2^{x(x-1)}$ and is invertible. Find $f^{-1}(x)$.

c) A matrix X has (a + b) rows and (a + 2) columns while the matrix Y has (b + 1) rows and (a + 3) columns. Both the matrices XY and YX exist. Find the values of a and b.

d) Fill in the blanks : If A and B' are independent events, then $P(A' \cup B) = 1 -$ __________.

Q08. a) Which equation of curve would satisfy $\dfrac{dy}{dx} = \sin(10x + 6y)$ such that it passes through origin?

b) Write the order and degree of the differential equation : $\left(\dfrac{d^2y}{dx^2}\right)^5 + \dfrac{4\left(\dfrac{d^2y}{dx^2}\right)^3}{\left(\dfrac{d^3y}{dx^3}\right)} + \left(\dfrac{d^3y}{dx^3}\right) = x^2 - 1$.

Q09. In a game, a man wins ₹10 for a number more than 4 and loses ₹3 for any other number, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he wins. Find the expected value of the amount he wins/loses. Do you think gambling is a good habit? Why?

Q10. Show that the equation of the perpendicular from the point (1, 6, 3) to the line $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$ is $\dfrac{x-1}{0} = \dfrac{y-6}{-3} = \dfrac{z-3}{2}$ and the foot of perpendicular is (1, 3, 5) and the length of the perpendicular is $\sqrt{13}$ units.

OR Find the foot of perpendicular of (1, 1, 0) in the plane $x + y - 2z = 3$.

Q11. A binary operation * on the set {0, 1, 2, 3, 4, 5} is defined as : $a*b = \begin{cases} a+b, & \text{if } a+b < 6 \\ a+b-6, & \text{if } a+b \geq 6 \end{cases}$.

Show that '0' is the identity element for this operation and each non-zero element 'a' of the set is invertible with '6 – a' being the inverse of 'a'.

Q12. Examine the continuity of the function $f(x)=\begin{cases}\dfrac{e^{1/x}-1}{e^{1/x}+1}, & x\neq 0\\ 0, & x=0\end{cases}$ at x = 0.

Q13. Evaluate : $\int \dfrac{x^2}{x^4-3x^2+16}dx$.

Q14. Differentiate $\tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}[2x\sqrt{1-x^2}]$, where $x\in\left(\dfrac{1}{\sqrt{2}},1\right)$.

Q15. Evaluate : $\int\limits_{-2}^{2}\dfrac{dx}{1+|x-1|}$. **OR** Evaluate : $\int\limits_{0}^{\pi/2}\sin 2x\tan^{-1}(\sin x)dx$.

Q16. Let $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$. Find a matrix D s. t. CD – AB = O where $C=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$.

Q17. Prove that : $\cos^{-1}\left(\dfrac{3}{5}\cos x+\dfrac{4}{5}\sin x\right)=\tan^{-1}\dfrac{4}{3}-x$, where $x\in\left[-\dfrac{3\pi}{4},\dfrac{\pi}{4}\right]$.

OR Solve for x : $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}(3x)$.

Q18. Show that the normal at any point θ to the curves $x=a\cos\theta+a\theta\sin\theta$, $y=a\sin\theta-a\theta\cos\theta$ is at a constant distance from the origin.

Q19. Show that $[\vec{a}\ \ \vec{b}\ \ \vec{c}]^2=\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c}\\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c}\\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c}\end{vmatrix}$. **OR** Show that $[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}]=0$.

SECTION – C

Q20. If $x+y+z=0$, prove that $\begin{vmatrix}xa & yb & zc\\ yc & za & xb\\ zb & xc & ya\end{vmatrix}=xyz\begin{vmatrix}a & b & c\\ c & a & b\\ b & c & a\end{vmatrix}$.

Q21. A point on the hypotenuse of a right angled triangle is at the distances *a* and *b* from the sides of the triangle. Show that the minimum length of hypotenuse is $(a^{2/3}+b^{2/3})^{3/2}$.

OR An open box with square base is to be made out of a given iron sheet of area 27 sq.metres, show that the maximum value of the volume of the box is 13.5 cubic metres.

Q22. Find the area of the region : $\left\{(x, y): |x+2|\le y\le\sqrt{5-x^2}\right\}$.

Q23. A bag contains 5 balls. Two balls are drawn at random from the bag and are found to be white. What is the probability that 4 balls in the bag are white and 1 is non-white?

Q24. Find the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})=0$ and $\vec{r}.(3\hat{i}+2\hat{j}+\hat{k})=0$. Show that this line is equally inclined to $\hat{i}$ and $\hat{k}$ and makes an angle $\dfrac{1}{2}\sec^{-1}(3)$ with $\hat{j}$.

Q25. A manufacturing company makes two type of teaching aids A and B of Mathematics of class XII. Each type of A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 hours, respectively. The profit on type A and B is ₹80 and ₹120 per piece, respectively. How many pieces of each type should be manufactured per week by the company to maximize its profit? What is the maximum profit per week?

Q26. Evaluate $\int\limits_{1}^{2}(4-e^x+x^2)dx$ as the limits of sums. **OR** Evaluate : $\int\limits_{0}^{1}\tan^{-1}(1-x+x^2)dx$.

Good Luck & Best Wishes!