MATHEMATICS
CLASS XII

Time: 3 hours **MM: 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of **26** questions divided into three sections **A**, **B** and **C**. Section **A** comprises **6** questions of **one mark** each, Section **B** comprises **13** questions of **four marks** each and Section **C** comprises 7 questions of **six marks** each.*
3. *All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.*
4. *There is no overall choice. However, internal choice has been provided in some questions of four marks and six marks . You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculator is not permitted. You may ask for logarithmic tables, if required.*

Section-A

Q1 For the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, find M_{12} and C_{23} where M_{12} is minor of the element in first row and second column and C_{23} is cofactor of the element in second row and third column. 1

Q2 Find the derivative of $\cos^{-1}(\sin x)$ $w.r.t\ \ x$ 1

Q3 Evaluate: $\int \left(x + \frac{1}{x} \right)^2 dx$. 1

Q4 Find the values of x, y, and z so that the vectors $\vec{a} = 2x\hat{i} + 3j + 2\hat{k}$ and $\vec{b} = 2\hat{i} + y\,j + z\hat{k}$ are equal. 1

Q5 Find the direction cosines of a line passing through the points (-1, 0, 2) and (3, 4, 6). 1

Q6 If a vector makes angles α, β, γ with x-axis, y-axis and z-axis respectively, then what is the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$. 1

Section-B

Q7 Solve the equation 4

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, x > 0$$

OR

Prove that
$$\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(x^2)$$

Q8 Using the properties of determinants, prove that 4

$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (a^3-1)^2$$

Q9 If $y = (\cos x)^{\log x} + (\log x)^x$. Find $\frac{dy}{dx}$. 4

Q10 Find the values of a and b so that the function 4

$$f(x) = \begin{cases} ax^2 + b & , \quad x < 2 \\ 2 & , \quad x = 2 \\ 2ax - b & , \quad x > 2 \end{cases}$$ may be continuous.

Q11 Find the equation of the tangent to the curve $y = \cos 2t,\ x = \sin 3t$ at $t = \frac{\pi}{4}$. 4

Q12 Evaluate: $\int_0^{\pi/2} \log \sin x dx$ 4

Q13 Solve the differential equation 4

$$y(1+e^x)dy = (y+1)e^x dx$$

OR

Solve the differential equation

$$x\frac{dy}{dx} = y + \sqrt{x^2 + y^2}$$

Q14 Solve the differential equation 4

$$x\frac{dy}{dx} + y = x\cos x + \sin x \text{ given that } y\left(\frac{\pi}{2}\right) = 1$$

Q15 If $\vec{a} = 2\hat{i} - 3\hat{j} + 5\hat{k}, \vec{b} = 4\hat{i} - 7\hat{j} + 2\hat{k}$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{c}$ and $\vec{a} \perp \vec{c}$, find $\vec{c}$ 4

Q16 A variable plane which remains at a constant distance of 9 units from the origin, cuts the coordinate axes at the points A, B and C. Show that the locus of the centroid of ΔABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{9}$ 4

OR

Find the foot of perpendicular from the point (2, 3, 4) to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also find the length of the perpendicular segment.

Q17 A car manufacturing factory has two plants. Plant P manufactures 70% of cars and plant Q manufactures 30%. At plant P, 80% of cars are rated of standard quality and at plant Q, 90% of cars are rated of standard quality. A car is picked up at random and is found to be of standard quality. What is the probability that it has come from plant P? 4

OR

If X follows binomial distribution with mean 3 and variance $\frac{3}{2}$, find P $(X \le 5)$

Q18 In a legislative assembly election, a political group hired a public relations firm to promote its candidate in 3 ways: telephone, house calls and letters. The cost per contact (in paise) is given by matrix A as 4

Cost per Contact

$$A = \begin{array}{cl} \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} & \begin{array}{c} \textit{Telephone} \\ \textit{House call} \\ \textit{Letter} \end{array} \end{array}$$

The number of contacts of each type made in two cities X and Y is given by

$$B = \begin{array}{cc} \begin{array}{ccc} \textit{Telephone} & \textit{House call} & \textit{Letter} \end{array} & \\ \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 1000 \end{bmatrix} & \begin{array}{c} \to X \\ \to Y \end{array} \end{array}$$

Find the total amount spent by the group in two cities X and Y, using matrix algebra.

Q19 Find the intervals in which the function $f(x) = 3x^4 - 16x^3 + 6x^2 + 72x$ is 4

(i) strictly increasing *(ii)* Strictly decreasing

Section-C

Q20 Let A = R – {2} and B = R – {1}. If f : A→B is a mapping defined by $f(x) = \dfrac{x-1}{x-2}$, show that f is a bijective. 6

Q21 Evaluate: $\int (3x-2)\sqrt{x^2+x+1}\, dx$. 6

Q22 using the integration, find the area of the triangular region whose vertices are P(1,0), Q(2,2) and R(3,1). 6

OR

Evaluate : $\int_1^4 (x^2 - x)dx$ as limit of a sum.

Q23 Find the volume of the largest cone that can be inscribed in a sphere of radius a cm. 6

OR

Find the points of local maxima/minima for the function

$$f(x) = \sin 2x - x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

Also find the local maximum and local minimum values.

Q24 3 bad eggs are mixed with 7 good ones. 3 eggs are taken at random from the lot. Find the probability distribution of number of bad eggs drawn. Find also the mean and variance of the probability distribution. 6

Q25 A manufacturer produes two products A and B. Both the products are processed on two different machines. The available capacity of the first machine is 12 hours and that of the second machine is 9 hours. Each unit of product A requires 3hours on both machines and each unit of product B requires 2hours on first machine and 1 hour on second machine. Each unit of product A is sold at a profit of Rs 5 and that of B at a profit of Rs 6.Find the production level for maximum profit graphically. 6

Q26 Determine whether or not the following pairs of lines intersect. If these intersect, find the point of intersection, otherwise obtain the shortest distance between them: 6

$$\vec{r} = \hat{i} + j + \lambda(3\hat{i} - j)$$

and $$\vec{r} = 4\hat{i} - \hat{k} + \mu(2\hat{i} + 3\hat{k})$$

Answers

Q1: $\begin{matrix} M_{12} = -46 \\ C_{23} = -13 \end{matrix}$ **Q2**: $\frac{d}{dx}\cos^{-1}(\sin x) = -1$ **Q3:** $= \frac{x^3}{3} - \frac{1}{x} + 2x + c$ **Q4**: x =1, y = 3, and z =2

Q5: Direction cosines of this line are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ **Q6**: 2 **Q7**: Part(a): $x = -\sqrt{3} + 2$

Q9: $\frac{dy}{dx} = (\cos x)^{\log x}\left[\frac{log(\cos x) - x\tan x \log x}{x}\right] + (\log x)^x\left[\frac{1 + \log x \log(\log x)}{\log x}\right]$

Q10: Function is continuous if $a = \frac{1}{2}$ and $b = 0$. **Q11**: $2\sqrt{2}x - 3y - 2 = 0$

Q12: $\int_0^{\pi} \log \sin x dx = -\frac{\pi}{2}\log 2$ **Q13**: Part(a) $\frac{e^y}{y+1} = c(1+e^x)$ OR Part(b) $cx^2 = y + \sqrt{x^2 + y^2}$

Q14: $y = \sin x$ **Q16**: Part(b): Length of perpendicular AP = $\frac{3\sqrt{101}}{7}$ **Q17:** 56/83 or $\frac{63}{64}$

Q18: X : Rs 3400; Y = Rs 2700

Q21: $= (x^2 + x + 1)^{3/2} - \frac{7}{8}(2x+1)\sqrt{x^2 + x + 1} - \frac{21}{16}\log\left|\frac{2x + 1 + 2\sqrt{x^3 + x + 1}}{\sqrt{3}}\right| + c$

Q22: Part(a) Area of ΔPQR = $\frac{3}{2}$ sq units OR Part(b) $\frac{27}{2}$ **Q23**: Part(a) Volume = $\frac{32\pi a^3}{81}$ cu units OR

Part(b) Local maximum value = $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$, Local minimum value = $-\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

Q24: Probability distribution

X	0	1	2	3
P(X)	$\frac{7}{24}$	$\frac{21}{40}$	$\frac{7}{40}$	$\frac{1}{120}$

Mean = 0.9, Variance = 0.49

Q25: Maximum value of Z is at C(0,6) **Q26**: Point of intersection is (4, 0, -1)