

SECTION-A

1. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of 'x'.
2. Write the order and degree of the differential equation : $y = x \frac{dy}{dx} + a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.
3. Write the integrating factor for solving the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right) \frac{dx}{dy} = 1$
4. Find the value of 'p' for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} + 2p\hat{j} + 3\hat{k}$ are parallel.
5. Find the area of the parallelogram determined by one diagonal $\hat{i} + 2\hat{j} + 3\hat{k}$ and one side $3\hat{i} - 2\hat{j} + \hat{k}$
6. Write the distance of a point P(a,b,c) from x-axis.

SECTION-B

7. Prove that : $2 \tan^{-1} \frac{1}{5} + \sec^{-1} \frac{5\sqrt{2}}{7} + 2 \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$

OR

Solve for 'x': $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \cos ecx)$

8. Find the matrix A if $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

9. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{bmatrix}$, Find A^{-1} by elementary transformations.

10. Solve for 'x': $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$ OR Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

11. Show that the function $f(x) = |x-3|$ is continuous at $x = 3$ but not differentiable at $x = 3$

12. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$.

OR

If $x = \sin t$, $y = \sin pt$ prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$.

13. Using Lagrange's mean value theorem find the points on the curve $y = x^3$ where the tangent is parallel to the chord joining (1,1) and (3,27).

14. Evaluate $\int \left(\log(\log x) + \frac{1}{(\log x)^2} \right) dx$.

15. Evaluate: $\int \frac{\sin x}{\cos x(\sin x + 1)} dx$ OR Evaluate: $\int x\sqrt{1+x-x^2} dx$

16. Evaluate: $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$.

17. Find the equation of the line through the point (1,-1,1) and perpendicular to the lines joining the points (4,3,2), (1,-1,0) and (1,2,-1), (2,1,1).

18. If \vec{a} , \vec{b} and \vec{c} are non-coplanar, Prove that $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are non-coplanar.

19. Four defective bulbs are accidentally mixed with six good ones. If it is not possible to just look at a bulb and tell whether or not it is defective, find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.

SECTION-C

20. Let $A = \{1, 2, 3, \dots, 12\}$ and R be a relation on A , defined as $R = \{ (a, b) : \text{both } a \text{ and } b \text{ are either odd or even} \}$. Show that R is an equivalence relation and hence write its equivalence classes.

21. Find the area of the triangle formed by positive x -axis, and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$, using integration.

OR

Evaluate $\int_0^2 (x + e^x) dx$ as the limit of a sum.

22. There are three coins. One is a biased coin that comes up with tail 60% of the times, the second is also a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it showed heads. What is the probability that it was the unbiased coin?

23. Find the local maximum and local minimum values of the function

$$f(x) = \sin^4 x + \cos^4 x, 0 < x < \pi.$$

OR

Find the values of 'x' for which $f(x) = [x(x-2)]^2$ is an increasing function. Also find the points where the tangents are parallel to the x-axis.

24. Show that the differential equation: $2y e^{\frac{x}{y}} dx + \left(y - 2x e^{\frac{x}{y}} \right) dy = 0$ is homogeneous and find

its particular solution, given that $x = 0$ when $y = 1$.

25. Find the equation of the plane through the intersection of the planes $x + 3y + 6z = 0$ and $3x - y - 4z = 0$ and whose perpendicular distance from origin is unity.

26. A village has 500 hectares of land to grow two types of crops, X and Y. The contribution of total amount of oxygen produced by X and Y are 60% and 40% per hectares. To control weeds, a liquid herbicide has to be used for X and Y at rates of 20 liters and 10 liters per hac.res. Further not more than 8000 liters of herbicide should be used to protect aquatic animals in a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximize the total production of Oxygen? How do you think access use of herbicides affects our environment?
