MATHEMATICS
CLASS XII

Time: 3 hours **MM: 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of **26** questions divided into three sections **A**, **B** and **C**. Section **A** comprises **6** questions of **one mark** each, Section **B** comprises **13** questions of **four marks** each and Section **C** comprises 7 questions of **six marks** each.*
3. *All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.*
4. *There is no overall choice. However, internal choice has been provided in some questions of four marks and six marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculator is not permitted. You may ask for logarithmic tables, if required.*

Section-A

Q1	If A = $\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $\|2A\| = 4\|A\|$.	1
Q2	If $f(x) = a^x \cdot x^a$ find $f'(x)$.	1
Q3	Evaluate: $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}}$.	1
Q4	The position vectors of points A, B, C and D are $\vec{a}$, $\vec{b}$, $2\vec{a}+3\vec{b}$ and $\vec{a}-2\vec{b}$. Express $\overrightarrow{DB}$ and $\overrightarrow{AC}$ in terms of $\vec{a}$ and $\vec{b}$.	1
Q5	Show that the vectors $3\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}-\hat{j}-\hat{k}$ are perpendicular.	1
Q6	A line makes an angle of $\frac{\pi}{4}$ with each of x-axis and y-axis. Find the angle between this line and the z-axis.	1

Section-B

Q7	Evaluate: $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3$ OR Solve the equation: $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{6}{17}\right)$	4
Q8	Using the properties of determinants, prove that $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \beta+\alpha \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$	4
Q9	If $y=\tan^{-1}\left[\frac{\sqrt{a}-\sqrt{x}}{1+\sqrt{ax}}\right]$. Find $\frac{dy}{dx}$.	4

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Q10	Find the interval in which the function $f(x)=\sin\left(2x+\frac{\pi}{4}\right), 0\le x\le 2\pi$ is *(i)* increasing *(ii)* decreasing	4
Q11	Evaluate: $\int\frac{x^2+1}{x^4+x^2+1}dx$ OR Evaluate : $\int\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$	4
Q12	Show that $y=Ax+\frac{B}{x}$ is a solution of the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$	4
Q13	Solve the differential equation $x^2dy+y(x+y)dx=0$ given that $y=1$ when $x=1$.	4
Q14	Find the angle between the line $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$ and the plane $3x+4y+z+5=0$ OR Find the equation of the plane containing the line of intersection of the planes $x+2y+3z-4=0$ and $2x+y-z+5=0$ and which is perpendicular to the plane $5x+3y+6z+8=0$.	4
Q15	Find a vector whose magnitude is 3 units and which is perpendicular to the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}=3\hat{i}+j-4\hat{k}$ and $\vec{b}=6\hat{i}+5j-2\hat{k}$.	4
Q16	An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the second urn. Find the probability that it is a white ball.	4
Q17	Let A = $\begin{bmatrix}0 & -\tan\left(\frac{\theta}{2}\right)\\ \tan\left(\frac{\theta}{2}\right) & 0\end{bmatrix}$. Show that I+A=(I-A)$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$.	4
Q18	If $f(x)$, defined by the following is continuous at $x=0$, find the values of a, b, c; $f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x}, & if \quad x<0\\ c, & if \quad x=0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{\frac{3}{2}}}, & if \quad x>0\end{cases}$	4
Q19	In a legislative assembly election, a political group hired a public relations firm to promote its candidate in 3 ways: telephone, house calls and letters. The cost per contact (in paise) is given by matrix A as	4

	Cost per Contact A= $\begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix}$ *Telephone* *House call* *Letter* The number of contacts of each type made in two cities X and Y is given by *Telephone* *House call* *Letter* B= $\begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 1000 \end{bmatrix} \begin{matrix} \rightarrow X \\ \rightarrow Y \end{matrix}$ Find the total amount spent by the group in two cities X and Y, using matrix algebra.	

Section-C

Q20	let $\oplus$ be a binary operation on Q defined by $a \oplus b = \frac{ab}{4}$. Show that the operation $\oplus$ is commutative as well as associative. Also find ts identity element and inverse of an element.	6
Q21	Evaluate: $\int (x+1)\sqrt{1-x-x^2}\, dx$. OR Evaluate the definite integral $\int_0^{\pi/2} \frac{\cos x}{1+\sin x+\cos x} dx$	6
Q22	In a test an examinee either guesses or copies or knows the answer to a multiple choice questionwith four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied it is $\frac{1}{8}$. Find the probability that he he knows the answer to the question given that h correctly answered it.	6
Q23	Show that the height of a right circular cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.	6
Q24	Using the definite integrals, find the area of the region bounded by the triangle whose vertices are (-1, 1), (0, 5) and (3, 2).	6
Q25	Prove that the lines $\vec{r} = \hat{i} + 2j + 3\hat{k} + \lambda(2\hat{i} + 3j + 4\hat{k})$ and $\vec{r} = 2\hat{i} + 3j + 4\hat{k} + \mu(3\hat{i} + 4j + 5\hat{k})$ intersect. Also find the vector equation of the plane in which they lie.	6
Q26	A diet for a sick person must contain atleast 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods X and Y are available at a cost of Rs 4 and Rs 3 per unit respectively. One unit of food X contains 200 units of vitamins, 1 units of minerals and 40 calories while one unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of foods X and Y should be used to have least cost, satisfy the requirements.	6

Answers

Q2: $a^x x^{a-1}(a + x\log a)$ **Q3:** $\frac{2}{3(a-b)}\left[(x+a)^{3/2} - (x+b)^{3/2}\right] + c$ **Q4**: $\overrightarrow{DB} = 3\vec{b} - \vec{a}$ and $\overrightarrow{AC} = \vec{a} + 3\vec{b}$

Q6: $\frac{\pi}{2}$ **Q7**: Part(a): π or Part(b): $x = \frac{1}{3}$ **Q9**: $\frac{-1}{2\sqrt{x}(1+x)}$ **Q10**: Part(a): Increasing : $\left[0, \frac{\pi}{8}\right]$ and $\left[\frac{5\pi}{8}, 2\pi\right]$, Decreasing : $\left[\frac{\pi}{8}, \frac{5\pi}{8}\right]$ OR Part(b): 7.04 **Q11**: Part(a): $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{3}}\right) + c$ OR Part(b): $2x\tan^{-1}x - \log(1+x^2) + c$ **Q13:** $y + 2x = 3x^2 y$ **Q14**: Part(a): $\sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)$ OR Part(b): $51x + 15y - 50z + 173 = 0$ **Q15**: Part(a) $2\hat{i} - 2j + \hat{k}$ OR $-2\hat{i} + 2j - \hat{k}$

Q16: $\frac{59}{130}$ **Q18**: $a = \frac{-3}{2}, c = \frac{1}{2}$, $b \in \text{R-}\{0\}$ **Q19**: X : Rs 3400; Y = Rs 2700

Q20: Identity element is 4. **Q21**: Part(a): $-\frac{1}{3}(1-x-x^2)^{3/2} + \frac{2x+1}{8}\sqrt{1-x+x^2} + \frac{5}{16}\sin^{-1}\left(\frac{2x+1}{\sqrt{5}}\right) + c$ Or Part(b): $\frac{\pi}{4} - \frac{1}{2}\log 2$ **Q22**: $\frac{24}{29}$ **Q24**: $\frac{15}{2}$ sq units

Q25: $\frac{3\sqrt{2}}{2}$ units **Q26**: X : 5 units, Y : 30 units

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