Time: 2:30 hour **Mock Board Exam Series-III**
M.M. 100 **Class XII** **By- Sandeep Shishodia**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consist of **26** questions divided into three sections A, B and C. Section A comprises of **6** questions of one mark each, section B comprises of **13** questions of four marks each and section C comprises of **7** questions of six marks each.*
3. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
4. *There is no overall choice. However, internal choice has been provided in **4** questions of four marks each and **2** questions of six mark each. You have to attempt only one of the alternatives in all such questions.*
5. *Please note this question paper consist **4** printed pages.*
6. *Use of calculators is not permitted. You may ask for logarithmic tables, if required.*

Section-A

1. Write the antiderivative of $\frac{2^x}{1+4^x}$ w.r.t. 'x'.
2. If $|\vec{a}|=a$, then find the value of $|\vec{a}\times\hat{i}|^2+|\vec{b}\times\hat{j}|^2+|\vec{c}\times\hat{k}|^2$.
3. If $A=\begin{bmatrix}1527 & 8365 & 92\\ 26 & 7382 & 8\\ 77020 & 728 & 90\end{bmatrix}$ and $|adjA|=|A|^k$, then find k.
4. If $f:\mathbf{R}\to\mathbf{R}$ be defined by $f(x)=\left(3-x^3\right)^{1/3}$, then find $fof(x)$.
5. *Evaluate* $\int \frac{|x}{y^{\circ}}\,dy$.
6. Write the Integrating Factor for the linear differential equation $\left(y^2-1\right)+2xy.\frac{dy}{dx}=\frac{2}{y^2-1}\frac{dy}{dx}$.

Section-B

7. Evaluate: $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$.

8. Prove that for any three vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$, $\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = 2\left[\vec{a}\vec{b}\vec{c}\right]$.

9. Vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$. Find the angle between $\vec{a}$ and $\vec{b}$.

10. Find the value of $\sin^{-1}\left[\cot\left\{\sin^{-1}\sqrt{\left(\frac{2-\sqrt{3}}{4}\right)} + \cos^{-1}\frac{\sqrt{12}}{4} + \sin^{-1}\frac{1}{\sqrt{2}}\right\}\right]$.

OR

Prove that $4\tan^{-1}\left[\frac{1}{5}\right] - \tan^{-1}\left[\frac{1}{70}\right] + \tan^{-1}\left[\frac{1}{99}\right] = \frac{\pi}{4}$.

11. Using properties of determinants prove that:

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2).$$

12. Find the value of a and b if $f(x) = \begin{cases} -2\sin x & x \le -\frac{\pi}{2} \\ a\sin x + b & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x & x \ge \frac{\pi}{2} \end{cases}$ is continuous at $x = \pm\frac{\pi}{2}$.

13. If $y = \sin^{-1}\left(\frac{4\sin x + 3\cos x}{5}\right)$ show that $\frac{dy}{dx} = 1$.

OR

If $x^2 - xy + y^2 = a^2$ show that $\frac{d^2y}{dx^2} = \frac{6a^2}{(x-2y)^3}$.

14. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function in $\left[0, \frac{\pi}{2}\right]$.

15. Evaluate : $\int \sqrt{1+\cot x}\, dx$. *How andwhy entropy of youth is integrating day by day? Give two points only.*

OR

Evaluate: $\int \cos^{-1}(\sin x)\, dx$ *.How andwhy entropy of youth is integrating day by day? Give two points only.*

16. Solve : $\boldsymbol{x\frac{dy}{dx} = y(\log x - \log y + 1)}$.

17. A bag contains 4 balls. Two balls are drawn at random, and are found to be blue. What is the probability that 50% balls were blue in colour in that bag?

Ball is associated in most of the outdoor and indoor games, how one can enhance maths learning by playing games. Give in brief.

18. Find the value of *x, y, z* if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$, satisfy the equation $A'A = I$.

19. If plane $x - y + z = 7$ is $4\sqrt{3}$ units far from point $(-3, 5, \lambda)$, then what is the value of λ?

OR

Show that the lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{2} = \frac{y-2}{-1} = \frac{-z+1}{-3}$ are lying on a plane/are coplanar.

Section-C

20. Find the area of the circle $x = \sqrt{16 - y^2}$ which is exterior to the parabola $y^2 = 6x$, using integration.

OR

Find the area between the curve $(x-1)^2 = 4y$ and $(x-2)^2 = 4 - y$ using integration.

21. A card being lost from a deck of 52 playing card. From the remaining cards two card drawn and found both were diamond cards. Find the probability that the lost card was a card other than diamond.

22. Solve the differential equation $x\frac{dy}{dx} + y - x + xy\cot x = 0$; $x \neq 0$

23. Given two matrix $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ *and* $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ verify that $BA = 6I$. Use the result to solve the system: $\begin{aligned} x - y &= 3; \\ 2x + 3y + 4z &= 17; \\ y + 2z &= 7. \end{aligned}$

24. If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, find the equations of Plane (s) containing these two lines.

OR

Find the equation of plane that passing through the line of intersection of planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$.

25. Find the equation of tangents to the curve $y = \cos(x + y)$, $-2\pi \le x \le 2\pi$ that are parallel to the line $x + 2y = 0$.

OR

A rectangular sheet of width 1 m is folded in such a way that its one corner is touching the edge of other side find the length of minimum size of creases.

26. The manager of an oil refinery must decide on the optimal mix of two possible blending processes of which the inputs & outputs per production run, are as follows:

	Input		Output	
Process	Crude A	Crude B	Gasoline P	Gasoline Q
1	5	3	5	8
2	4	5	4	4

The max. Crude A& B available are 200 & 150 units resp. Market requirements are at least 100 & 80 units P & Q respectively .The profit from process 1 & process 2 are Rs.300/- & Rs 400/- resp. Formulate LPP & solve for maximising the profit. A greedy petrol pump owner is mixing kerosene in petrol. What value you will suggest him to stop such crime on your level best.