MATHEMATICS
CLASS XII

Time: 3 hours **MM: 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of **26** questions divided into three sections **A**, **B** and **C**. Section **A** comprises **6** questions of **one mark** each, Section **B** comprises **13** questions of **four marks** each and Section **C** comprises 7 questions of **six marks** each.*
3. *All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.*
4. *There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculator is not permitted. You may ask for logarithmic tables, if required.*

Section-A

Q1	If A = $\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that |2A|= 4|A|.	1
Q2	If $f(x) = a^x . x^a$ find $f'(x)$.	1
Q3	Evaluate: $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}}$.	1
Q4	The position vectors of points A, B, C and D are $\vec{a}$, $\vec{b}$, $2\vec{a}+3\vec{b}$ and $\vec{a}-2\vec{b}$. Express $\overrightarrow{DB}$ and $\overrightarrow{AC}$ in terms of $\vec{a}$ and $\vec{b}$.	1
Q5	Show that the vectors $3\hat{i} + j + 2\hat{k}$ and $\hat{i} - j - \hat{k}$ are perpendicular.	1
Q6	A line makes an angle of $\frac{\pi}{4}$ with each of x-axis and y-axis. Find the angle between this line and the z-axis.	1

Section-B

Q7	Evaluate: $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$ OR Solve the equation: $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{6}{17}\right)$	4
Q8	Using the properties of determinants, prove that $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\alpha & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$	4
Q9	If $y = \tan^{-1}\left[\frac{\sqrt{a}-\sqrt{x}}{1+\sqrt{ax}}\right]$. Find $\frac{dy}{dx}$.	4

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Q10	Find the interval in which the function $f(x)=\sin\left(2x+\frac{\pi}{4}\right), 0\le x\le 2\pi$ is *(i)* increasing *(ii)* decreasing	4
Q11	Evaluate: $\int \frac{x^2+1}{x^4+x^2+1}dx$ OR Evaluate : $\int \sin\left(\frac{2x}{1+x^2}\right)dx$	4
Q12	Show that $y=\mathrm{A}x+\frac{\mathrm{B}}{x}$ is a solution of the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$	4
Q13	Solve the differential equation $x^2dy+y(x+y)dx=0$ given that $y=1$ when $x=1$.	4
Q14	Find the angle between the line $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$ and the plane $3x+4y+z+5=0$ OR Find the equation of the plane containing the line of intersection of the planes $x+2y+3z-4=0$ and $2x+y-z+5=0$ and which is perpendicular to the plane $5x+3y+6z+8=0$.	4
Q15	Find a vector whose magnitude is 3 units and which is perpendicular to the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}=3\hat{i}+\hat{j}-4\hat{k}$ and $\vec{b}=6\hat{i}+5\hat{j}-2\hat{k}$.	4
Q16	An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the second urn. Find the probability that it is a white ball.	4
Q17	Let A = $\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$.Show that I+A=(I-A)$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.	4
Q18	Discuss the continuity of the following function at $x=0$; $f(x)=\begin{cases} \frac{x^4+2x^3+x^2}{\tan^{-1}x}, & x\neq 0 \\ 0, & x=0 \end{cases}$	4
Q19	Find the equations of tangents to the curve $y=x^3+2x+6$ which are perpendicular to the line $x+14y+4=0$. OR If the tangent to the curve $y=x3+ax+b$ at P (1,-6) is parallel to the line $y-x=5$, find the values of a and b.	4

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Section-C

Q20	let $\oplus$ be a binary operation on Q defined by $a \oplus b = \frac{ab}{4}$. Show that the operation $\oplus$ is commutative as well as associative. Also find is identity element and inverse of an element.	6
Q21	Evaluate: $\int (x+1)\sqrt{1-x-x^2}\ dx$. OR Evaluate the definite integral $\int_0^{\pi/2} \frac{\cos x}{1+\sin x+\cos x} dx$	6
Q22	In a test an examinee either guesses or copies or knows the answer to a multiple choice questionwith four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied it is $\frac{1}{8}$. Find the probability that he he knows the answer to the question given that h correctly answered it.	6
Q23	Show that the height of a right circular cylinder that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. OR A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 m. Find the dimension of the window to admit maximum light through the whole opening.	6
Q24	Using the definite integrals, find the area of the region bounded by the triangle whose vertices are (-1, 1), (0, 5) and (3, 2).	6
Q25	Prove that the lines $\vec{r} = \hat{i} + 2j + \hat{k} + \lambda(2\hat{i} + 3j + 4\hat{k})$ and $\vec{r} = 2\hat{i} + 3j + 4\hat{k} + \mu(3\hat{i} + 4j + 5\hat{k})$ intersect. Also find the vector equation of the plane in which they lie.	6
Q26	A diet for a sick person must contain atleast 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods X and Y are available at a cost of Rs 4 and Rs 3 per unit respectively. One unit of food X contains 200 units of vitamins, 1 units of minerals and 40 calories while one unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of foods X and Y should be used to have least cost, satisfy the requirements.	6

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Answers

Q2: $a^x x^{a-1}(a+x\log a)$ **Q3:** $\frac{2}{3(a-b)}\left[(x+a)^{3/2}-(x+b)^{3/2}\right]+c$ **Q4**: $\overrightarrow{DB}=3\vec{b}-\vec{a}$ and $\overrightarrow{DC}=\vec{a}+3\vec{b}$

Q6: $\frac{\pi}{2}$ **Q7**: Part(a): π or Part(b): $x=\frac{1}{3}$ **Q9**: $\frac{-1}{2\sqrt{x}(1+x)}$ **Q10**: Part(a): Increasing : $\left(0,\frac{3\pi}{8}\right)$ and $\left(\frac{5\pi}{8},2\pi\right)$, Decreasing : $\left(\frac{3\pi}{8},\frac{5\pi}{8}\right)$ OR Part(b): 7.04 **Q11**: Part(a): $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{3}}\right)+c$ OR Part(b): $2x\tan^{-1}x-\log(1+x^2)+c$ **Q13:** $y+2x=3x^2y$ **Q14**: Part(a): $\sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)$ OR Part(b): $51x+15y-50z+173=0$ **Q15**: Part(a) $2\hat{i}-2\hat{j}+\hat{k}$ OR $-2\hat{i}+2\hat{j}-\hat{k}$

Q16: $\frac{59}{130}$ **Q18**: Continuous **Q19**: $14x-y-10=0;\ 14x-y+22=0$ OR (2,-4); $\left(\frac{-2}{3},\frac{4}{27}\right)$

Q20: Identity element is 4. **Q21**: Part(a): $-\frac{1}{3}(1-x-x^2)^{3/2}+\frac{2x+1}{8}\sqrt{1-x+x^2}+\frac{5}{16}\sin^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)+c$

Or Part(b): $\frac{\pi}{4}-\frac{1}{2}\log 2$ **Q22**: $\frac{59}{130}$ **Q23:** Length= $\frac{10}{4+\pi}$ meters, breadth= $\frac{20}{4+\pi}$ meters= Diameter of semi-circular portion.

Q24: $\frac{15}{2}$ sq units **Q25**: $\frac{3\sqrt{2}}{2}$ units **Q26**: X : 5 units, Y : 30 units

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