HALF SYLLABUS TEST – 03 B

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Max. Marks : 60 **Time Allowed : 2 Hours**

Section A

Q01. An edge of variable cube is increasing at the rate of 3cm/s. How fast is the volume of the cube increasing when the edge is 10cm long?

Q02. If '^' is a binary operation which is defined as "a ^ b = $a^2 + 2b$" then, determine the value of 3 ^ 2.

Q03. Evaluate : $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx$.

Q04. Write the value of $\int_{-\pi/2}^{\pi/2} \log\left|\frac{2-\sin x}{2+\sin x}\right| dx$.

Q05. Write the integrating factor of differential equation : $(1 + y^2)\,dx = (\tan^{-1}y - x)\,dy$.

Q06. If $x \in R$, $0 \le x \le \frac{\pi}{2}$, and $\begin{vmatrix} 2\sin x & -1 \\ 1 & \sin x \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ -4 & \sin x \end{vmatrix}$, then find the values of x.

Section B

Q07. For what value of k, $f(x) = \begin{cases} \frac{\log(1+ax) - \log(1-bx)}{x}, & if\ x \ne 0 \\ k, & if\ x = 0 \end{cases}$ is continuous at x = 0?

Q08. If $x = a(t - \sin t)$, $y = a(1 - \cos t)$, find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$. **OR** Differentiate $x^{x\cos x} + \frac{x^2+1}{x^2-1}$ w.r.t. x.

Q09. Evaluate $\int_1^3 (x^2 + x)dx$ as the limit of a sum.

Q10. Evaluate : $\int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$.

Q11. Let $f : N \to N$ be defined as $f(n) = \begin{cases} \frac{n+1}{2}, & \text{when } n \text{ is odd} \\ \frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$ for all $n \in N$.

State whether the function f is bijective. Justify your answer.

Q12. Solve : $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$.

Section C

Q13. Prove that the volume of largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

Q14. Evaluate : $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$. **OR** Evaluate : $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2\sin^2 x} dx$.

Q15. Using properties of determinants, prove that : $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$.

Q16. Use transformations to find inverse of $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

Q17. Prove that : $\int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x})dx = \sqrt{2}\,\pi$.