Max. Marks : 85
Time : 120 Minutes
Topics : Chapter 01, 08 – 13 of NCERT Textbooks (Based on 2015 Exams.)

HST – 02 A

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NOTE : (i) All the questions are compulsory. Internal choices has been provided in a few sums.
(ii) This paper is based on CBSE Board Exams 2015 of various regions viz. Delhi, All India (various regions) and Foreign.

SECTION – A *(Each question carry One mark.)*

Q01. The equations of a line are $5x-3=15y+7=3-10z$. Write the direction cosines of the line.

Q02. Find the sum of the order and degree of the given differential equation : $\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\}=0$.

Q03. In a triangle OAC, if B is the mid-point of the side AC and $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$, then what is $\overrightarrow{OC}$?

Q04. Find a vector of magnitude $\sqrt{171}$ which is perpendicular to both of the vectors $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$.

Q05. Find the angle between the lines $2x=3y=-z$ and $6x=-y=-4z$.

Q06. Write the integrating factor of $(1+y^2)+(2xy-\cot y)\frac{dy}{dx}=0$.

Q07. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $|2\hat{a}+\hat{b}+\hat{c}|$.

SECTION – B *(Each question carry Four marks.)*

Q08. For 6 trials of an experiment, let X be a binomial variate which satisfies the relation $9\,P(X=4)=P(X=2)$. Find the probability of success.

Q09. Find the shortest distance between the following lines :
$\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ and $\vec{r}=2\hat{i}+4\hat{j}+5\hat{k}+\mu(4\hat{i}+6\hat{j}+8\hat{k})$.

OR Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$ and $5x-3y+4z+9=0$ and is parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{5-z}{-5}$.

Q10. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d}=27$.

Q11. A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of 5 steps, he is one step away from the starting point.

OR Suppose a girl throws a die. If she gets a 1 or 2, she tosses a coin three times and notes the number of 'tails'. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die?

Q12. Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spades. Hence find the mean of the distribution

Q13. Find the equation of a line passing through the point (1, 2,–4) and perpendicular to two lines $\vec{r}=(8\hat{i}-19\hat{j}+10\hat{k})+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$ and $\vec{r}=(15\hat{i}+29\hat{j}+5\hat{k})+\mu(3\hat{i}+8\hat{j}-5\hat{k})$.

OR Find the equation of the plane passing through the points (–1, 2, 0), (2, 2, –1) and parallel to the line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.

SECTION – C *(Each question carry Six marks.)*

Q14. Let N denote the set of all natural numbers and R be the relation on N×N defined by (a,b)R(c,d) if $ad(b+c)=bc(a+d)$. Show that R is an equivalence relation.

Q15. If the area bounded by the parabola $y^2 = 16ax$ and the line $y = 4mx$ is $\frac{a^2}{12}$ sq.units, then using integration, find the value of m.

Q16. Determine whether the relation R defined on the set R of all real numbers as $R = \{(a,b): a, b \in R$ and $a - b + \sqrt{3} \in S$, where S is the set of all irrational numbers$\}$, is reflexive, symmetric and transitive.

OR Let $A = R \times R$ and * be the binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Prove that * is commutative and associative. Find the identity element for * on A. Also write the inverse element of the element (3, –5) in A.

Q17. **(a)** Show that the differential equation $(x - y)\frac{dy}{dx} = x + 2y$ is homogeneous and solve it also.

(b) Find the differential equation of the family of curves $(x - h)^2 + (y - k)^2 = r^2$, where h and k are arbitrary constants.

Q18. Consider $f : R_+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible, hence find f^{-1}.

Q19. An urn contains 5 red and 2 black balls. Two balls are randomly drawn, without replacement. Let X represent the number of black balls drawn. What are the possible values of X? Is X a random variable? If yes, find the mean and variance of X.

OR In a factory which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their output 3, 4 and 1 percent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

Q20. A company manufactures three kinds of calculators : A, B and C in its two factories I and II. The company has got an order for manufacturing at least 6400 calculators of kind A, 4000 of kind B and 4800 of kind C. The daily output of factory I is of 50 calculators of kind A, 50 calculators of kind B, and 30 calculators of kind C. The daily output of factory II is of 40 calculators of kind A, 20 of kind B and 40 of kind C. The cost per day to run factory I is ₹12000 and of factory II is ₹15000. How many days do the two factories have to be in operation to produce the order with the minimum cost? Formulate this as an LPP and solve it graphically.

Q21. Find the equation of a plane passing through the point P(6, 5, 9) and parallel to the plane determined by the points A(3, –1, 2), B(5, 2, 4) and C(–1, –1, 6). Also find the distance of this plane from the point A.

OR Find the distance of the point P(3, 4, 4) from the point, where the line joining the points $A(3, -4, -5)$ and B(2, –3, 1) intersects the plane $2x + y + z = 7$.

Q22. A binary operation * is defined on the set $X = R - \{-1\}$ by $x * y = x + y + xy, \forall x, y \in X$. Check whether * is commutative and associative. Find the identity element and also find the inverse of each element of X.