RELATIONS AND FUNCTIONS

1 Mark

1) If $f(x) = \frac{x-1}{x+1}, (x \neq 1, -1)$, show that fof^{-1} is an identity function.

2) If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\Pi}{3}\right) + \cos x . \cos\left(x + \frac{\Pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then find the value of $gof(x) = x$

(Ans : 1)

3) Let * be a binary operation on the set of real numbers. If a * b = a+b-ab, 2 * (3 * x) = -7, find the value of x. (Ans : x=-2)

4) Find the number of One-One functions from a finite set A to A, where n(A) = P (Ans : P!)

5) Let A = {4,5,0}. Find the number of binary operations that can be defined on A. (Ans: 3^9)

6) Let $\int :\{R \rightarrow R\}$ be a function defined by $f(x) = x^2-3x+4$, for all $x \varepsilon R$, find the value of $f^{-1}(2)$

Ans: {1,2}

7) Let $f : R \rightarrow R$ defined by $f(x) = (ax^2+b)^3$ find the function g : $R \rightarrow R$ such that f(g(x)) = g(f(x))

Ans: $\left[\frac{x^{\frac{1}{3}} - b}{a}\right]^{\frac{1}{2}}$

8) If $f(x) = \frac{5x+3}{4x-5}\left(x \neq \frac{5}{4}\right)$, find g(x) such that gof(x) = x Ans: $g(x) = \frac{5x+3}{4x-5}$

9) If $f(x) = \frac{1+x}{1-x}$, show that $f[f(\tan\theta)] = -\cot\theta$

10) Show that $\frac{1}{\sin^3 x} + \cot x + \frac{1}{x^5} + x^3$ is an odd function.

11) Let f, g be two functions defined by

$f(x) = \frac{x}{x+1}, g(x) = \frac{x}{1-x}$, then find $(fog)^{-1}(x)$ Ans: x

12) Let $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$, find the value of α such that f(f(x)) = x Ans: $\alpha = -1$

4 Marks / 6 marks

13) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, show that $f(x) + f(y) = f\left(\frac{x+y}{1+xy}\right)$

14) If R is a relation on a set $A(A \neq \phi)$, prove that R is symmetric iff $R^{-1} = R$

15) Show that the relation R on N x N defined by $(a,b)R(c,d) \Leftrightarrow$ a+d = b+c is an equivalence relation.

16) Let $f : R_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2+6x-5$. Show that f is invertiable with $f^{-1}(y) = \frac{\sqrt{y+6} - 1}{3}$

17) Show that the relation "congruence modulo 2" on the set Z (set of integers) is an equivalence relation. Also find the equivalence class of 1.

18) If the function $f : R \to A$ given by $f(x) = \frac{x^2}{x^2+1}$ is surjection, find the set A.

Ans: A = Range of f(x)=[0,1)

19) Let a relation R on the set R of real numbers be defined as (a,b) $\varepsilon R \Leftrightarrow 1+ab > 0$ for all $a, b\ \varepsilon\ R$. show that R is reflexive and symmetric but not transitive.

20) Let a relation R on the set R of real numbers defined as $(x,y)\varepsilon R \Leftrightarrow x^2 4xy + 3y^2 = 0$. Show that R is reflexive but neither symmetric nor transitive.

21) Let N denote the set of all natural numbers and R be the relation on N x N defined by $(a,b)R(c,d) \Leftrightarrow ad(b+c) = bc(a+d)$. Show that R is an equivalence relation on N x N.

22) Prove that the inverse of an equivalence relation is an equivalence relation.

23) Let $f : A \to B$ be a given function. A relation R in the set A is given by $R = \{(a,b)\ \varepsilon\ A \times A : f(a) = f(b)\}$. Check, if R is an equivalence relation. Ans: Yes

24) Let f and g be real valued functions, such that (fog)(x) = $\cos x^3$ and (gof)(x) = $\cos^3 x$, find the functions f and g. Ans: f(x)=cosx, g(x) = x^3

25) Define a binary operation * on the set A = {0,1,2,3,4,5} as

$$a * b = \begin{cases} a+b, & \text{if } a+b < 6 \\ a+b-6, & \text{if } a+b \geq 6 \end{cases}$$

Show that zero is an identity element for this operation and each element a of the set is invertiable with 6-a being the inverse of a.

26) Show that the function $f : N \to N$ given by f(x) = x - $(-1)^x$ for all $x\ \varepsilon\ N$ is a bijection.

27) Prove that relation R defined on the set N of natural numbers by

x R y $\Leftrightarrow$ $2x^2-3xy+y^2 = 0$ is not symmetric but it is reflexive.

28) Let X = {1,2,3,4,5,6,7,8,9}. Let R_1 and R_2 be relations in X given by

R_1 = {(x,y) : x-y is divisible by 3} and

R_2 = {(x,y) ! {x,y} ε {1,4,7} or {x,y} ε {2,5,8}

or {x,y} ε {3,6,9}}. Show that $R_1 = R_2$

29) Determine which of the following functions

f : R $\to$ R are (a) One - One (b) Onto

(i) f(x) = |x| + x

(ii) f(x) = x - [x]

(Ans: (i) and (ii) $\to$ Neither One-One nor Onto)

30) On the set N of natural numbers, define the operation * on N by m*n = gcd(m,n) for all m, n ε N. Show that * is commutative as well as associative.

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