MATHEMATICS
CLASS XII
Relation&Function,Matrices and Deteminants

Time: 3 hours **MM: 100**

1. *The question paper consists of 26 questions divided into three sections A, B and C.*
2. *Section A contains 6 questions of 1 mark each, Section B contains 13questions of 4 marks each and Section C contains 7 questions of 6 marks each.*
3. *All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.*
4. *Use of calculator is not permitted. You may ask for logarithmic tables, if required.*

SECTION – A

Q1 Let f, g : R $\to R$ be defined by f(x) = 2x + 1 and g(x) = $x^2 - 2$, $\forall$ x $\in$R, respectively. Then find go$f$. 1

Q2 If the binary operation * on the set Z of integers is defined by a* b = a + $3b^2$, find the value of 2*4. 1

Q3 State the reason for the relation R on the set {1,2,3} given by R ={ (1,2), (2,1)} not to be transitive. 1

Q4 If $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$, then write the value of k. 1

Q5 Find the value of x + y from the following equation: 1

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Q6 A is a square matrix of order 3 and $|A| = 7$. Write the value of $|adjA|$. 1

SECTION – B

Q7 Show that the relation R on the set A={1,2,3,4,5}, given by R = {(a,b) : | a – b | is even }, is an equivalence relation. Show that all the elements of { 1,3,5} are related to each other and all the elements of { 2,4} are related to each other. But no element of { 1,3,5} is related to any element of { 2,4}. 4

Q8 Let f : N $\to R$ be a function defined as f(x) = $4x^2 + 12x + 15$. Show that f :N $\to$ Range (f) is invertible. Find the inverse of f. 4

Q9 Find the value of x, such that 4

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$$

Q10 Express the following matrix as the sum of a symmetric and skew symmetric matrix and verify your result $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$. 4

Q11 If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then verify that A adj A = $|A|I$. 4

Q12 Find the matrix A such that 4

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

Q13 Consider the binary operation '*' on the set{ 1,2,3,4,5} defined by a * b= min { a,b}. Write the operation table of the operation '*'. 4

Q14 Show that the matrix $\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is inverse of the matrix $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$. 4

Q15 A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide rs 30,000 among the two types of bonds. If the trust must obtain an annual total interest of : (a) Rs 1800 (b) Rs 2000 4

Q16 Two schools A and B decide to award prizes to their students for three values honesty (x), punctuality (y) and obedience (z). School A desires to award a total Rs 11000 for the three values to 5,4 and 3 students respectively while school B wishes to award R s 10700 for the values to 4,3 and 5 students respectively . If all the three prizes together amount to Rs 2700, then: 4

(i) Represent the above situation by a matrix equation and form linear equations using matrix multiplication.

(ii) Is it possible to solve the system of equations so obtained using matrices?

(iii) Which value you prefer to be rewarded most and why?

Q17 Solve for x, the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0.$ 4

Q18 Using properties of determinants, prove the following : 4

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab+bc+ca+abc.$$

Q19 If a,b,c are positive and are respectively equal to the pth , qth and rth terms of a G.P., then show that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0.$ 4

SECTION - C

Q20 If A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ *and* $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equations : x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7. 6

Q21 If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\left(\frac{\alpha}{2}\right) & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that 6

$$I + A = (I - A)\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}.$$

Q22 Obtain the inverse of the following matrix using elementary operation $A=\begin{bmatrix}0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1\end{bmatrix}_{3\times 3}$ 6

Q23 The sum of three numbers is 20. Three times the first number added to the sum of second and third is 46. Twice the third number added to the first is 23. Find the numbers using matrices. 6

Q24 Prove that $\begin{vmatrix}3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c\end{vmatrix}=3(a+b+c)(ab+bc+ca).$ 6

Q25 Find the values of θ satisfying the equation $\begin{vmatrix}1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2\end{vmatrix}=0.$ 6

Q26 If $A=\begin{bmatrix}2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2\end{bmatrix}$ Verify that $A^3-6A^2+9A-4I=O$ and hence find A^{-1}. 6