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PLEASURE TEST SERIES XII – 16

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Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. If $f : R \to R$, $g : R \to R$ be defined by $f(x) = \frac{3x-7}{8}$, $g(x) = \frac{8x+7}{3}$ then, find $fog(7)$.

Q02. Find the cofactor of a_{12} in $\begin{vmatrix} 2 & 3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

Q03. Evaluate $\int_0^{3/2} [x]dx$, where [.] represents a greatest integer function.

Q04. Find the tangent of the angle between $\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$.

Q05. Write the interval in which $f(x)$ is continuous where $f(x) = e^x \log |x|$.

Q06. Find the centre and radius of the circle, if possible, represented by $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2}$.

SECTION B

Q07. Prove that : $\tan^{-1}[x + \sqrt{1+x^2}] = \frac{\pi}{4} + \frac{1}{2}\tan^{-1} x,\ x \in R$.

OR Simplify the expression : $\frac{1}{2}\cos^{-1}\left(\frac{5\cos x + 3}{5 + 3\cos x}\right)$.

Q08. Using properties, prove that : $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{a^2+c^2}{b} \end{vmatrix} = 4abc$.

Q09. Determine the value of k for which $f(x) = \begin{cases} 2x+1, & \text{if } x < 2 \\ k, & \text{if } x = 2 \\ 3x-1, & \text{if } x > 2 \end{cases}$ is continuous at x = 2.

Q10. Find the equation of normal to curve $x = \sin 3t,\ y = \cos 2t$ at $t = \pi/4$.

Q11. If x = a sin2t (1 + cos2t) and y = b cos2t (1 – cos2t), show that $\left(\frac{dy}{dx}\right)_{\text{at } t = \pi/4} = \frac{b}{a}$.

Q12. Find the equation of a curve passing through the point (1, 1) if the perpendicular distance of the origin from the normal at any point P(x, y) of the curve is equal to the distance of P from the x-axis.

Q13. Using properties of definite integrals, evaluate : $\int_0^{\pi/2} \frac{\sec^2 x}{\sec^2 x + \operatorname{cosec}^2 x} dx$.

Q14. Find the particular solution of the differential equation : $(2y + x)dy - (2y - x)dx = 0,\ y(1) = 1$.

OR Solve the differential equation : $(x^2 - y^2)dx + 2xydy = 0$ given that $y = 1$ when $x = 1$.

Q15. Find a point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of $3\sqrt{2}$ units from the point (1, 2, 3).

Q16. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$ then, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$.

OR Decompose the vector $6\hat{i} - 3\hat{j} - 6\hat{k}$ into the vectors which respectively are parallel and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$.

Q17. Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is an invertible function. Hence find f^{-1}.

Q18. Evaluate : $\int \sqrt{\frac{x}{a^3 - x^3}} dx$.

Q19. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes.

OR If P (A) = 0.4, P (B) = p, $P(A \cup B) = 0.6$ and A and B are given to be independent events, find the value of p.

SECTION C

Q20. Find the area of the region bounded by the curve $y = x^2 + x$, x-axis and the line x = 2 and x = 5.

OR Find the area enclosed by the curve x = 3 cos t, y = 2 sin t.

Q21. A school has to reward the students participating in co-curricular activities (Category I), with 100% attendance (Category II) and brave students (Category III) in a function. The sum of the numbers of all the three category students is 6. If we multiply the number of students of category III by 2 and add to the number of students of category I to the result, we get 7. By adding II and III category students to three times the I category students, we get 12. Form the matrix equation and, hence solve it. Which value has been shown here?

Q22. Show that the height of a cylinder of maximum volume which can be inscribed in a cone of height h and semi vertical angle α is h/3.

OR Find the maximum area of a rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of major axis.

Q23. Evaluate : $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}} dx$.

Q24. Three families of four children consists of 3 girls, 1 boy; 2 girls, 2 boys and, 1 girls, 3 boys respectively. If one children is selected from each family, then what is the probability that it comprises of one girl and two boys?

Q25. A retired person has ₹70,000 to invest on two types of bonds available in the market. Bond I yields an annual income of 8% on the amount invested and the Bond II yields 10% per annum. As per norms, he has to invest a minimum of ₹10,000 in Bond I and not more than ₹30,000 in Bond II. How should he plan his investment, so as to get maximum return, after one year of investment?

Q26. Find the equation of the plane which passes through the points (3, 4, 1) and (0, 1, 0) and is parallel to the line $\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$.

ANSWERS *Of* PTS XII – 16 [2015-16]

Q01. We have $fog(7) = f[g(7)] = f\left(\frac{8\times7+7}{3}\right) = f(21) = \frac{3\times21-7}{8} = 7$.

Q02. Cofactor of $a_{12} = -[(6)(-7)-(4)(1)] = 46$.

Q03. We have $\int_0^{3/2}[x]dx = \int_0^1[x]dx + \int_1^{3/2}[x]dx = \int_0^1 0dx + \int_1^{3/2}1dx = 0 + [x]_1^{3/2} = \frac{3}{2} - 1 = \frac{1}{2}$.

Q04. Using dot product of vectors, $\cos\theta = \frac{|(\hat{i}-\hat{j}+\hat{k}).(\hat{i}+\hat{j}-\hat{k})|}{\sqrt{3}\sqrt{3}} = \frac{1}{3} \Rightarrow \tan\theta = 2\sqrt{2}$.

Q05. R – {0} **Q06.** (0, 0) & 1 unit.

Q07. Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$ …(i) in LHS. So, let $Y = \tan^{-1}[x+\sqrt{1+x^2}] = \tan^{-1}[\tan\theta + \sqrt{1+\tan^2\theta}]$

$$\Rightarrow Y = \tan^{-1}[\tan\theta + \sec\theta] = \tan^{-1}\left(\frac{1+\sin\theta}{\cos\theta}\right) = \tan^{-1}\left(\frac{\cos^2(\theta/2)+\sin^2(\theta/2)+2\sin(\theta/2)\cos(\theta/2)}{\cos^2(\theta/2)-\sin^2(\theta/2)}\right)$$

$$\Rightarrow Y = \tan^{-1}\left(\frac{1+\tan(\theta/2)}{1-\tan(\theta/2)}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right) = \frac{\pi}{4}+\frac{\theta}{2} \Rightarrow Y = \frac{\pi}{4}+\frac{1}{2}\tan^{-1}x = \text{RHS.} \quad \text{[using (i)}$$

OR $\tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right)$ (See Mathematicia Vol. 1).

Q08. See Mathematicia Vol. 1

Q09. Since *f* is continuous at x = 2, so LHL (at x = 2) = RHL (at x = 2) = $f(2)$ …(i)
Now here $f(2) = k$ …(ii)
LHL (at x = 2) : $\lim_{x\to2^-} 2x+1 = 2(2)+1 = 5$ …(iii)
By (i), (ii) and (iii), k = 5.

Q10. Eq. of normal at $t = \pi/4$: $y - \cos 2(\pi/4) = \frac{3\cos3(\pi/4)}{2\sin2(\pi/4)}\left[x - \sin3(\pi/4)\right]$

$$\Rightarrow y - 0 = \frac{-3\times\frac{1}{\sqrt{2}}}{2\times1}\left[x - \frac{1}{\sqrt{2}}\right] \qquad \Rightarrow 3\sqrt{2}\,x + 4y = 3.$$

Q11. We have x = a sin2t (1 + cos2t) = 2a sin2t $\cos^2$t $\Rightarrow \frac{dx}{dt} = 2a\left(-\sin^2 2t + 2\cos^2 t\cos2t\right)$

And y = b cos2t (1 – cos2t) = 2b cos2t $\sin^2$t $\Rightarrow \frac{dy}{dt} = 2b\left(\cos2t\sin2t - 2\sin^2 t\sin2t\right)$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx} = \frac{2b\left(\cos2t\sin2t - 2\sin^2 t\sin2t\right)}{2a\left(-\sin^2 2t + 2\cos^2 t\cos2t\right)}$$

So, $\left(\frac{dy}{dx}\right)_{\text{at } t=\pi/4} = \frac{b\left(\cos2(\pi/4)\sin2(\pi/4) - 2\sin^2(\pi/4)\sin2(\pi/4)\right)}{a\left(-\sin^2 2(\pi/4) + 2\cos^2(\pi/4)\cos2(\pi/4)\right)} = \frac{b}{a}$. Hence proved.

Q12. $x^2 + y^2 - 2x = 0$ (Example 08 in NCERT Exemplar Problems, Chapter 09)

Q13. Use $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ to get the value of integral as $\frac{\pi}{4}$.

Q14. We have $(2y+x)dy - (2y-x)dx = 0 \Rightarrow \frac{dy}{dx} = \frac{2y-x}{2y+x}$. Put $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So we get : $v + x\frac{dv}{dx} = \frac{2vx-x}{2vx+x} \Rightarrow \int\frac{(2v+1)dv}{2v^2-v+1} = -\int\frac{dx}{x} \Rightarrow \frac{1}{2}\int\frac{(4v-1)dv}{2v^2-v+1} + \frac{3}{2}\int\frac{dv}{2v^2-v+1} = -\int\frac{dx}{x}$

$$\Rightarrow \frac{1}{2}\log|2v^2 - v + 1| + \frac{3}{4}\int \frac{dv}{\left(v - \frac{1}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2} = -\int \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\log\left|2\left(\frac{y^2}{x^2}\right) - \frac{y}{x} + 1\right| + \frac{3}{4} \times \frac{4}{\sqrt{7}}\tan^{-1}\left(\frac{4v-1}{\sqrt{7}}\right) = -\log|x| + C$$

$$\Rightarrow \log\left|2y^2 - xy + x^2\right| - 2\log|x| + \frac{6}{\sqrt{7}}\tan^{-1}\left(\frac{4y-x}{x\sqrt{7}}\right) = -2\log|x| + 2C$$

$$\Rightarrow \log\left|2y^2 - xy + x^2\right| + \frac{6}{\sqrt{7}}\tan^{-1}\left(\frac{4y-x}{x\sqrt{7}}\right) = K, \text{ where } K = 2C.$$

Given that y = 1 when x = 1 so, $\log 2 + \frac{6}{\sqrt{7}}\tan^{-1}\left(\frac{3}{\sqrt{7}}\right) = K$

Hence the solution is : $\Rightarrow \log\left|2y^2 - xy + x^2\right| + \frac{6}{\sqrt{7}}\tan^{-1}\left(\frac{4y-x}{x\sqrt{7}}\right) = \log 2 + \frac{6}{\sqrt{7}}\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$

OR We have $2\frac{dy}{dx} = \frac{y^2 - x^2}{xy}$. It is homogenous so put y = vx and get : $x^2 + y^2 - 2x = 0$.

Q15. Let P(1, 2, 3). The coordinates of random point on $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = m$ is M(3m –2, 2m – 1, 2m + 3). So PM = $3\sqrt{2} \Rightarrow m = 0, 30/17$. $\therefore$ Required points : $(-2, -1, 3)$ and $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$.

Q16. $\vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$ [See Solutions of CBSE 2013 Delhi (set 2)]

OR See Mathematicia Vol. 2

Q17. We have $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$.

To test whether *f* is one-one: Let $x_1, x_2 \in R$ and let $f(x_1) = f(x_2) \Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0 \Rightarrow 3(x_1 - x_2)[3(x_1 + x_2) + 2] = 0$

$\Rightarrow (x_1 - x_2) = 0$ $[\because x_1, x_2 \in R_+ \Rightarrow x_1 + x_2 \neq 0 \Rightarrow 3(x_1 + x_2) + 2 \neq 0]$

$\Rightarrow x_1 = x_2$. $\therefore$ The function *f* is one-one.

To test whether *f* is onto: Let y be any arbitrary element of $[-5, \infty)$.

Let $y = 9x^2 + 6x - 5$ $\Rightarrow y = (3x+1)^2 - 6$ $\Rightarrow (3x+1)^2 = y + 6$

$\Rightarrow (3x+1) = \sqrt{y+6}$ $[\because y \geq -5 \Rightarrow y + 6 \geq 0]$ i.e., $\Rightarrow x = \frac{\sqrt{y+6} - 1}{3}$.

Clearly $x \in R_+$ for all $y \in [-5, \infty)$. Therefore, *f* is onto.

Hence, the function is invertible. And, $f^{-1} = \frac{\sqrt{y+6} - 1}{3}$.

Q18. Let $I = \int \sqrt{\frac{x}{a^3 - x^3}}\, dx = \int \frac{\sqrt{x}}{\sqrt{a^3 - (x^{3/2})^2}}\, dx$. Put $x^{3/2} = t \Rightarrow \frac{3}{2}\sqrt{x}\, dx = dt$

$$\therefore I = \frac{2}{3}\int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}} = \frac{2}{3}\sin^{-1}\left(\frac{t}{a^{3/2}}\right) + C \quad \Rightarrow I = \frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + C.$$

Q19. Let X : number of doublets in 4 throws of dice. So X = 0, 1, 2, 4.

X	0	1	2	3	4
P(X)	625/1296	500/1296	150/1296	20/1296	1/1296

OR See Mathematicia Vol. 2.

Q20. Construct a labeled and clean diagram.

Required area = $\int_2^5 (x^2+x)dx = \frac{99}{2}$ Sq.units.

OR 6π Sq.units

Q21. x + y + z = 6, x + 2z = 7, 3x + y + z = 12 where x, y, z represent the number of students in categories I, II, III respectively. Also x = 3, y = 1, z = 2. Participating in co-curricular activities is very important. It is very essential for all round development.

Q22. See Mathematicia Vol. 1 **OR** See Mathematicia Vol. 1

Q23. Let $I = \int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\,dx = \int_{-a}^{a} \frac{a}{\sqrt{a^2-x^2}}\,dx - \int_{-a}^{a} \frac{x}{\sqrt{a^2-x^2}}\,dx$.

Show that first integral is an even function whereas second is an odd function, therefore by using

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, \text{ if } f(x) \text{ is even function } i.e., f(-x)=f(x) \\ 0, \text{ if } f(x) \text{ is odd function } i.e., f(-x)=-f(x) \end{cases}$$

We have $I = 2\int_0^a \frac{a}{\sqrt{a^2-x^2}}\,dx - 0 = 2a\left[\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a = a\pi$.

Q24. Case I : P (one girl from family I, one boy from family II, one boy from family III)
Case II : P (one girl from family II, one boy from family III, one boy from family I)
Case III : P (one girl from family III, one boy from family I, one boy from family II)

Thus, required probability $= \frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{2}{4}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}\times\frac{2}{4}=\frac{26}{64}$.

Q25. See Q31 Chapter 08 in OP Gupta's MATHEMATICIA Vol.1

Q26. Let A, B, C be d.r.'s of normal of the plane passing through (3, 4, 1) and (0, 1, 0).
So the plane is A (x – 3) + B(y – 4) + C(z – 1) = 0 …(i)
As (i) passes through (0, 1, 0) i.e., – 3A – 3B – C = 0 i.e., 3A + 3B + C = 0 …(ii)
Also plane is parallel to $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$ therefore, 2A + 7B + 5C = 0 …(iii)
By (ii) and (iii), we get $\frac{A}{8}=\frac{B}{-13}=\frac{C}{15}$ i.e., d.r.'s of normal are 8, –13, 15.
Replacing these values in (i) : $8(x-3)-13(y-4)+15(z-1)=0$ i.e., 8x – 13y + 15z + 13 = 0.