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Series : PTS/13

Code No. 15/11/13

Candidates must write the Code on the title page of the answer-book.

Roll No. | | | | | | | |

PLEASURE TEST SERIES XII – 13

A Compilation By : O. P. Gupta [Call or WhatsApp @ +91-9650 350 480]

Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. The position vectors of points A and B are $\vec{a}$ and $\vec{b}$ respectively. P divides AB in the ratio 3 : 1 and Q is mid-point of AP. Find the position vector of Q.

Q02. Find the area of the parallelogram, whose diagonals are $\vec{d}_1 = 5\hat{i}$ and $\vec{d}_2 = 2\hat{j}$.

Q03. If P(2, 3, 4) is the foot of perpendicular from origin to a plane, then write the vector equation of this plane.

Q04. If $\Delta = \begin{vmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{vmatrix}$, write the cofactor of a_{32} (the element of third row and 2nd column).

Q05. If m and n are the order and degree, respectively of the differential equation

$$y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$$, then write the value of m + 2n.

Q06. Write the differential equation representing the curve $y^2 = 4ax$, where a is an arbitrary constant.

SECTION B

Q07. A part of the monthly expenses of a family is constant while the remaining varies with the price of rice, fuel etc., When the price of rice is Rs.25/Kg the monthly expenses of the family is Rs.1000. when it is Rs.24/Kg the monthly expenses is Rs.980. Find the total monthly expenses of the family when the cost of rice is Rs 35/Kg. Is this family below poverty line? Give two suggestions to improve their standard of living.

Q08. If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that A satisfies the equation $A^3 - 4A^2 - 3A + 11I = O$.

OR If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix}$, find A^{-1} using elementary row operations.

Q09. If x, y, z are in GP, then using properties of determinants, show that

$$\begin{vmatrix} px+y & x & y \\ py+z & y & z \\ 0 & px+y & py+z \end{vmatrix} = 0$$, where $x \neq y \neq z$ and p is any real number.

Q10. Evaluate : $\int_{-1}^{1} |x \cos \pi x| \, dx$.

Q11. Evaluate : $\int \frac{1+\sin 2x}{1+\cos 2x} e^{2x} dx$. **OR** Evaluate : $\int \frac{x^4}{(x-1)(x^2+1)} dx$.

Q12. Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 3' given that 'there is at least one head'.

OR How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

Q13. For three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ if $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \times \vec{c} = \vec{b}$, then prove that $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, $|\vec{b}| = |\vec{a}|$ and $|\vec{a}| = 1$.

Q14. Find the equation of the line through the point (1, –1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, –1, 0) and (1, 2, –1), (2, 1, 1).

OR Find the position vector of the foot of perpendicular drawn from the point P(1, 8, 4) to the line joining A(0, –1, 3) and B(5, 4, 4). Also find the length of this perpendicular.

Q15. Evaluate : $\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos(\tan^{-1}\sqrt{3})$. **OR** Prove that : $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$.

Q16. If x = sin t, y = sin kt, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + k^2 y = 0$.

Q17. Differentiate $\tan^{-1}\frac{\sqrt{1-x^2}}{x}$ with respect to $\cos^{-1}[2x\sqrt{1-x^2}]$, where $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$.

Q18. It is given that for the function f (x) = $x^3 + bx^2 + ax + 5$ on [1, 3], Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b.

Q19. Evaluate : $\int \frac{1+3x}{\sqrt{5-2x-x^2}} dx$.

SECTION C

Q20. Let A = {1, 2, 3, …, 10} and R be the relation in A × A defined by (a, b) R (c, d) if a + d = b + c for a, b, c, d in A × A.

Prove that R is an equivalence relation. Also obtain the equivalence class [(3, 7)].

OR Let f : N ⟶ R be a function defined as f (x) = $4x^2 + 12x + 15$.

Show that f : N ⟶ S is invertible, where S is the range of f. Hence find inverse of f.

Q21. Compute, using integration, the area bounded by the lines x + 2y = 2, y – x = 1 and 2x + y = 7.

Q22. Find the particular solution of the differential equation $xe^{y/x} - y\sin\left(\frac{y}{x}\right) + x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) = 0$, given that $y = 0$, when $x = 1$.

OR Obtain the differential equation of all circles of radius r.

Q23. Show that the lines $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 5\hat{k})$ and $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$ are coplanar. Also, find the equation of the plane containing these lines.

Q24. 40% students of a college reside in hostel and the remaining reside outside. At the end of year, 50% of the hosteliers got A grade while from outside students, only 30% got A grade in the examination. At the end of year, a student of the college was chosen at random and was found to get A grade. What is the probability that the selected student was a hostelier?

Q25. A man rides his motorcycle at the speed of 50km/h. He has to spend Rs. 2 per km on petrol. If he rides it at a faster speed of 80km/h, the petrol cost increases to Rs. 3 per km. He has atmost Rs. 120 to spend on petrol and one hour's time. Using LPP find the maximum distance he can travel.

Q26. A jet of enemy is flying along the curve y = $x^2 + 2$ and a soldier is placed at the point (3, 2). Find the minimum distance between the soldier and the jet.