<u>**PRE-BOARD EXAMINATION**</u>

<u>**MATHEMATICS**</u>

STD: XII **M.M.: 100**

DATE: **TIME: 3 HRS**

General Instructions:

1. All questions are compulsory.

2. The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, section B comprises of 13 questions of four marks each and section C comprises of 07 questions of six marks each.

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, internal choice has been provided in 04 questions four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

<u>**SECTION - A**</u>

(1) If $\tan^{-1}\alpha + \tan^{-1}\beta = \frac{\pi}{4}$, then write the value of $\alpha + \beta + \alpha\beta$. **(1)**

(2) If $\vec{x}$ is a unit vector such that $\vec{x} \times \hat{i} = \vec{k}$, find $\vec{x}.\hat{j}$. **(1)**

(3) If $\begin{vmatrix} \sin\theta & -\cos\theta \\ \sin\delta & \cos\delta \end{vmatrix} = \frac{\sqrt{3}}{2}$; where θ, δ are acute angles, then write the value of $\theta + \delta$. **(1)**

(4) If $y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots \ldots\ldots \infty}}}$ then find $\frac{dy}{dx}$. **(1)**

(5) Let * be a binary operation on N given by a*b = HCF (a, b), a, b, ∈ N, write 44 * 6. **(1)**

(6) Evaluate: $\int_{-1}^{1} \frac{|x|}{x} dx$. **(1)**

<u>**SECTION – B**</u>

(7) Show that the function f: R → R defined by $f(x) = x^3 + 3$ is invertible. Find f^{-1}. Hence find $f^{-1}(-5)$ and find $f^{-1}(30)$. **(4)**

(8) Prove that: $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\tan^{-1}\left(2\sqrt{2}\right)$. **(4)**

(9) Show that the equation $(x + y)\,dy + (x - y)\,dx = 0$ is homogeneous. Also find the particular solution given that x = 1 when y = 1. **(4)**

OR

Solve the differential equation: $y + x\sin\left(\frac{y}{x}\right) = x\frac{dy}{dx}$.

(10) The surface area of a spherical bubble is increasing at the rate of 2 cm^2/sec. Find the rate at which the volume of the bubble is increasing at the instant, when radius is 6 cm. **(4)**

OR

Find the equations of tangents to the curve $3x^2 - y^2 = 8$, which pass through the point (4/3, 0).

(11) If $y = e^{a\cos^{-1}x}$, $-1 \le x \le 1$, show that $(1 - x^2)y_2 - xy_1 - a^2y = 0$. **(4)**

(12) Evaluate: $\int x(\log x)^2 dx$. **(4)**

(13) Find the image of the point having position vector $\hat{i} + 3\hat{j} + 3\hat{k}$ in the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$. **(4)**

(14) Determine the constants p and q, such that the function **(4)**

$$f(x) = \begin{cases} px^2 + q, if\ x > 2 \\ 2, \quad if\ x = 2 \\ 2px - q. if\ x < 2 \end{cases}$$ is continuous.

OR

Show that the function $f(x) = |x - 3|$, is continuous but not differentiable at x = 3.

(p.t.o)

(15) Prove that $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$. **(4)**

(16) Differentiate $\sin x^2$ *with respect to* $e^{\cos x}$. **(4)**

(17) In a group of 50 NCC cadets, 30 are well trained in shooting technique, while the remaining are well trained in disaster management. Two cadets are selected from the group. Find the probability distribution of the number of selected cadets who are well trained in disaster management. Find the mean of the distribution also. Write one more value which is expected from a sell trained cadet. **(4)**

(18) Find the area of the circle $x^2 + y^2 = 6x$, lying above x – axis and bounded by the parabola $y^2 = 3x$. **(4)**

(19) For non zero vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$, show that $\vec{a} - \vec{b}$, $\vec{b} - \vec{c}$ *and* $\vec{c} - \vec{a}$ are always coplanar. **(4)**

OR

Dot product of a vector with vectors $\hat{\imath} - \hat{\jmath} + \hat{k}$, $2\hat{\imath} + \hat{\jmath} - 3\hat{k}$ and $\hat{\imath} + \hat{\jmath} + \hat{k}$ are respectively 4, 0 and 2. Find the vector.

SECTION – C

(20) Solve the following system of equations using matrices: **(6)**
$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2; x, y, z \neq 0$.

OR

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations
$x - y + 2z = 1; 2y - 3z = 1; 3x - 2y + 4z = 2$.

(21) A given quantity of metal sheet is to be cast into a solid half circular cylinder with rectangular base and semicircular ends. Show that in order that the total surface area may be minimum the ratio of length of the cylinder to the diameter of its circular ends is $\pi : \pi + 2$. **(6)**

(22) In a group of 900 students, 200 attend extra classes, 300 attend school regularly and 400 students study themselves with the help of peers. The probability that the students will succeed in competition who attends extra classes, attend school regularly and study themselves with the help of peers is 0.3, 0.4 and 0.2 respectively. One student is selected, who succeeded in the competition. What is the probability that he attend school regularly? What the value depicted by student attend the school regularly? **(6)**

(23) Find the equation of the plane passing through the line of intersection of the planes containing the lines $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 1$ *and* $\vec{r} \cdot (2\hat{\imath} + 3\hat{\jmath} - \hat{k}) + 4 = 0$ and parallel to the *x*- axis. Also, find the distance of this plane from origin. **(6)**

(24) By using properties Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}}$ **(6)**

OR

Evaluate: $\int_0^2 (x^2 + x + 1)dx$, *as a limit of sum.*

(25) A manufacturer makes two products, A and B. Product A sells at RS. 200 each and takes ½ hours to make. Product B sells at Rs. 300 each and takes 1 hour to make. There is a permanent order for 14 units of product A and 16 units of product B. A working week consists of 40 hours of production and the weekly turnover must not be less than Rs. 10000. If the profit on each of product A is Rs.20 and on product B is Rs. 30, then how many of each should be produced so that the profit is maximum? Also find the maximum profit. **(6)**

(26) Find the distance of the point (2, 2, - 1) from the plane $x + 2y - z = 1$ measured parallel to the line $\frac{x+1}{2} = \frac{y+1}{2} = \frac{z}{3}$. **(6)**

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