CLASS – XII (Set–I)

MATHEMATICS

PRE BOARD-1

Time allowed: 3 hours **Maximum marks: 100**

General Instructions:

1. *All questions are compulsory.*

2. *The question paper consists of 26 questions divided into three sections-A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13questions of four marks each and Section C comprises of 7 questions of six marks each.*

3. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

4. *There is no overall choice. However, internal choice has been provided in 3questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

5. *Use of calculators is not permitted.*

SECTION–A

1. If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then for what value of α is A an identity matrix?

2. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

3. Evaluate: $\int \frac{x^2}{1+x^3}\,dx$

4. Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

5. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$

6. Find the value of λ so that the lines

$$\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11} \text{ and } \frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}.$$

are perpendicular to each other.

SECTION–B

7. Prove that $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$.

8. Using elementary transformations, find the inverse of the following matrix:

$$\begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 1 \\ 1 & 6 & 2 \end{bmatrix}$$

9. Using properties of determinants, prove the following:

$$\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2\,(a+b)$$

10. In a Legislative assembly election, a political party hired a public relation firm to promote its candidate in three ways; telephone, house calls and letters. The numbers of contacts of each type in three cities A, B & C are (500, 1000, and 5000), (3000, 1000, 10000) and (2000, 1500,4000), respectively. The party paid Rs. 3700, Rs.7200, and Rs.4300 in cities A, B & C respectively. Find the costs per contact using matrix method. Keeping in mind the economic condition of the country, which way of promotion is better in your view?

11. Find the value of 'a' for which the function f defined as

$$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x+1), & x\le 0 \\ \dfrac{\tan x-\sin x}{x^3}, & x>0 \end{cases}$$

is continuous at $x=0$.

12. If $x^{16}\,y^{9}=(x^2+y)^{17}$, prove that $\frac{dy}{dx}=\frac{2y}{x}$.

13. If $x=a\left(\cos t+\log\tan\frac{t}{2}\right)$, $y=a\sin t$, find $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.

14. Find the intervals in which the function f given by

$$f(x)=\sin x+\cos x, \qquad 0\le x\le 2\pi$$

15. is s Solve the following differential equation:

Fin $\frac{dy}{dx}=\frac{x(2y-x)}{x(2y+x)}$, if $y=1$ when $x=1$ the tangent is equal to the

y-c

16. Find λ if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

17.

Find the shortest distance between the following two lines :

$$\vec{r} = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (\lambda+1)\hat{k};$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}).$$

18.

A man is known to speak truth 3 out of 4 times. He throws a die and report that it is a 6. Find the probability that it is actually 6.

19. Find the particular solution of the differential equation $(tan^{-1}y - x)dy = (1 + y^2)dx$ for x = 1 and y = $\frac{\pi}{4}$.

SECTION–C

20. Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by 4$\}$ is an equivalence relation. Find the set of all elements related to 1.

21.

Find the volume of the largest cylinder that can be inscribed in a sphere of radius r.

OR

A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs Rs. 70 per sq. metre for the base and Rs. 45 per sq. metre for sides, what is the cost of least expensive tank?

22.

Evaluate: $\int_0^{\pi/2} 2\sin x \cos x \tan^{-1}(\sin x)\, dx$

OR

Evaluate: $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

23.

Using integration, find the area of the following region:

$$\left\{(x, y) ; \frac{x^2}{9} + \frac{y^2}{4} \le 1 \le \frac{x}{3} + \frac{y}{2}\right\}$$

OR,

Using the method of integration, find the area of the region bounded by the following lines:

$5x - 2y - 10 = 0$

$x + y - 9 = 0$

$2x - 5y - 4 = 0$

24.

Find the equation of the plane passing through the point $(-1, -1, 2)$ and perpendicular to each of the following planes:

$2x + 3y - 3z = 2$ and $5x - 4y + z = 6$

OR

Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$, $\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

25.

A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹17.50 per package on nuts and ₹7 per package of bolts. How many packages of each should be produced each day so as to maximize his profits if he operates his machines for at the most 12 hours a day? Form the linear programming problem and solve it graphically.

26.

Three bags contain balls as shown in the table below:

Bag	Number of White balls	Number of Black balls	Number of Red balls
I	1	2	3
II	2	1	1
III	4	3	2

A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they came from the III bag?

CLASS – XII (Set–II)

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2. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

3. Evaluate: $\int \frac{x^2}{1+x^3}\,dx$

4. Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

5. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$

6. Find the value of λ so that the lines

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are perpendicular to each other.

SECTION–B

If $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a \sin t$, find $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.

7. Prove the following:

$$\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1}\left(\frac{1-x}{1+x}\right), x \in (0, 1)$$

8. Using elementary transformations, find the inverse of the following matrix:

$$\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$$

9. Using properties of determinants, prove the following:

$$\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2 (a+b)$$

10. Three shopkeepers A, B, C are using polythene, handmade bags (prepared by prisoners), and newspaper's envelope as carry bags. it is found that the shopkeepers A, B, C are using (20,30,40) , (30,40,20,) , (40,20,30) polythene , handmade bags and newspapers envelopes respectively. The shopkeepers A, B, C spent Rs.250, Rs.220 & Rs.200 on these carry bags respectively .Find the cost of each carry bags using matrices. Keeping in mind the social & environmental conditions, which shopkeeper is better? & why?

11. Find the value of 'a' for which the function f defined as

$$f(x) = \begin{cases} a \sin \frac{\pi}{2}(x+1), & x \le 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$$

is continuous at $x = 0$.

12. If $x^{13} y^7 = (x+y)^{20}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

13. Find the intervals in which the function f given by

$f(x) = \sin x + \cos x, \quad 0 \le x \le 2\pi$

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$$\frac{dy}{dx} = \frac{x(2y-x)}{x(2y+x)}, \text{ if } y = 1 \text{ when } x = 1$$

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15. Find λ if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

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$$\vec{r} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (\lambda + 1)\hat{k};$$

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A man is known to speak truth 3 out of 4 times. He throws a die and report that it is a 6. Find the probability that it is actually 6.

18.

19. Find the particular solution of the differential equation $(tan^{-1}y - x)dy = (1 + y^2)dx$ for x = 1 and y = $\frac{\pi}{4}$.

SECTION–C

20. Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by 4$\}$ is an equivalence relation. Find the set of all elements related to 1.

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25. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs. 300 and that on a chain is Rs 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically.

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