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MATHEMATICIA
ALL THE MATH GYAN
PATHFINDERS

SHARP SHOT TESTS SERIES 02

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Time Allowed : 180 Minutes *Max. Marks : 100*

SECTION – A

Q01. For the set A = {1, 2, 3}, define a relation R in the set A as follows :

R = {(1, 1), (2, 2), (3, 3), (1, 3)}.

Write the ordered pairs to be added to R to make it the smallest equivalence relation.

Q02. Evaluate : $\cos^{-1}\cos\frac{2\pi}{3}+\sin^{-1}\sin\frac{2\pi}{3}$.

Q03. Find $f'(x)$ if, $f(x)=|x|, x\neq 0$.

Q04. Find the maximum and minimum values (if any) of $f(x)=-|x+1|+3$ on R.

Q05. Write $\text{adj.}(\text{adj.}A)$ for $A=\begin{bmatrix}1 & 2 & 0\\ 0 & -1 & 0\\ 1 & 3 & 4\end{bmatrix}$.

Q06. Write the equation of plane which cuts off equal intercepts of unit length on the coordinate axes.

SECTION – B

Q07. Using properties, find the value of $\Delta=\begin{vmatrix}(x-2)^2 & (x-1)^2 & x^2\\ (x-1)^2 & x^2 & (x+1)^2\\ x^2 & (x+1)^2 & (x+2)^2\end{vmatrix}$.

Q08. If $y=\cot^{-1}\left(\frac{x^x-x^{-x}}{2}\right)$ then, find the value of $\frac{dy}{dx}$ at $x=1$.

OR If $xy^2=1$ then, show that $2\left(\frac{dy}{dx}\right)+y^3=0$.

Q09. Let $f, g : R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x \ \forall\ x \in R$. Then, find *fog* and *gof*.

Q10. Find the value(s) of p and q for which $f(x)=\begin{cases}\frac{\sin(p+1)x+\sin x}{x}, & x<0\\ q, & x=0\\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0\end{cases}$ is continuous for all x in R.

Q11. Use matrix multiplication to divide ₹30000 in two parts such that the total annual interest at 9% on the first part and 11% on the second part amounts ₹3060.

Q12. Write the differential equation of the family of circles with fixed radius 5 units and centre on the straight line $y = 2$.

Q13. Evaluate $\int_0^2 [x^2]dx$. **OR** Evaluate $\int_0^{\pi/2} \frac{\cos x}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}dx$.

Q14. Solve : $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$, $x>0$.

OR Prove that : $\cos^{-1}\left(\frac{\cos\alpha+\cos\beta}{1+\cos\alpha\cos\beta}\right)=2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$.

Q15. Solve : $\operatorname{cosec} x \log y \frac{dy}{dx}+x^2y^2=0$.

Q16. Evaluate : $\int \sqrt[3]{\frac{\sin^2 x}{\cos^{14} x}}dx$.

Q17. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is 5/6. What is the probability that he will knock down fewer than 2 hurdles?

Q18. Find the equations of the perpendicular drawn from the point (2, 4, –1) to the line $x+5=\frac{1}{4}(y+3)=-\frac{1}{9}(z-6)$ and hence obtain the coordinates of the foot of this perpendicular.

OR If the product of the distances of the point (1, 1, 1) from the origin and the plane $x-y+z+k=0$ be 5, then determine the value of k.

Q19. Solve the system of equations : $2x-y+3z=5,\ 3x+2y-z=7,\ 4x+5y-5z=9$.

SECTION – C

Q20. For two vectors $\vec{a}$ and $\vec{b}$, state and prove **Cauchy-Schwartz inequality**.

OR Prove that for any two vectors $\vec{a}$ and $\vec{b}$, we always have $|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$.

Q21. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500 L, 3000 L and 3500 L respectively. The distances (in km.) between the depots and the petrol pumps is given in the table shown below :

Distance (in km.)		
From/To	**A**	**B**
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is ₹ 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

Q22. Of all the closed right circular cylindrical cans of volume 128π cm^3, find the dimension of the can which has minimum surface area.

Q23. Find the area enclosed by $x^2+y^2-6x-4y+12 \le 0,\ y \le x$ and $x \le 5/2$.

Q24. If $x\cos\alpha+y\sin\alpha=p$ touches the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then prove that $a^2\cos^2\alpha+b^2\sin^2\alpha=p^2$.

OR Profit function of a company is given as $p(x)=\frac{24x}{5}-\frac{x^2}{100}-500$ where x is the number of units produced. What is the maximum profit of the company? Company feels its social responsibility and decided to contribute 10% of his profit for the orphanage. What is the amount contributed by the company for the charity? Should every company do it?

Q25. The probability of simultaneous occurrence of at least one of two events A and B is p. If the probability that exactly one of A, B occurs is q, then prove that P (A′) + P (B′) = 2 – 2p + q.

Q26. Find the equation of a plane passing through the intersection of planes $2x+3y-z+1=0$ and $x+y-2z+3=0$ and perpendicular to the plane $3x-y-2z=4$. Also find the inclination of this plane with xy-plane.

HINTS / ANSWERS / REFERENCES Of SSTS – 02

Q01. (3, 1). **Q02.** π. **Q03.** $f'(x)=\begin{cases}1, \text{ if } x>0\\ -1, \text{ if } x>0\end{cases}$.

Q04. Maximum value = 3 and minimum value isn't defined.

Q05. $-4A$. **Q06.** $x+y+z=1$. **Q07.** $\Delta=-8$ (See O.P. Gupta's Mathematicia Vol. 1).

Q08. $\dfrac{dy}{dx}=\dfrac{2x^x(1+\log x)}{1+x^{2x}}$ $\therefore \left.\dfrac{dy}{dx}\right]_{\text{at } x=1}=1$

OR $xy^2=1$ $\Rightarrow x=\dfrac{1}{y^2}$ $\Rightarrow \dfrac{dx}{dx}=\dfrac{d}{dx}\left(\dfrac{1}{y^2}\right)$ $\Rightarrow 1=\left(-\dfrac{2}{y^3}\right)\times\dfrac{dy}{dx}$

$\Rightarrow y^3=-2\dfrac{dy}{dx}$ $\therefore 2\left(\dfrac{dy}{dx}\right)+y^3=0$.

Q09. $fog(x)=\begin{cases}0, \text{ if } x\geq 0\\ -4x, \text{ if } x<0\end{cases}$ and, $gof(x)=0\ \forall\ x\in R$

Q10. $-3/2, 1/2$ **Q11.** ₹12000 and ₹18000. **Q12.** $(y-2)^2y_1^2=25-(y-2)^2$

Q13. Let $I=\int_0^2[x^2]dx$ $\Rightarrow I=\int_0^1[x^2]dx+\int_1^{\sqrt{2}}[x^2]dx+\int_{\sqrt{2}}^{\sqrt{3}}[x^2]dx+\int_{\sqrt{3}}^{2}[x^2]dx$

$\Rightarrow I=\int_0^1 0dx+\int_1^{\sqrt{2}}1dx+\int_{\sqrt{2}}^{\sqrt{3}}2dx+\int_{\sqrt{3}}^{2}3dx=5-\sqrt{2}-\sqrt{3}$

Note that here we have broken the given limits of x in such a way that on squaring, they results into two consecutive integers for x^2. [As in $\sqrt{2}<x<\sqrt{3}\ \Rightarrow 2<x^2<3$]

OR Let $I=\int_0^{\pi/2}\dfrac{\cos x}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}dx$ $\Rightarrow I=\int_0^{\pi/2}\dfrac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}dx$

$\Rightarrow I=\int_0^{\pi/2}\dfrac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}dx$. Then substitute $\cos\frac{x}{2}+\sin\frac{x}{2}=t$ and proceed to get : $I=2-\sqrt{2}$.

Q14. $x=2-\sqrt{3}$. **Q15.** $1+\log y=y\left(2\cos x+2x\sin x-x^2\cos x\right)+Cy$.

Q16. Let $I=\int\sqrt[3]{\dfrac{\sin^2 x}{\cos^{14}x}}dx$ $\Rightarrow I=\int\dfrac{\sin^{2/3}x}{\cos^{14/3}x}\times\dfrac{\cos^4 x}{\cos^4 x}dx$ $\Rightarrow I=\int\dfrac{\sin^{2/3}x}{\cos^{2/3}x}\times\dfrac{1}{\cos^4 x}dx$

$\Rightarrow I=\int\tan^{2/3}x\sec^4 x\,dx$. Then proceed further to get : $I=\dfrac{3}{5}\tan^{5/3}x+\dfrac{3}{11}\tan^{11/3}x+C$.

Q17. $\dfrac{5^{10}}{2\times 6^9}$ **Q18.** $(-4, 1, -3)$, $\dfrac{x-2}{6}=\dfrac{y-4}{3}=\dfrac{z+1}{2}$. **OR** $k=4,-6$.

Q19. See O.P. Gupta's Mathematicia Vol.1.

Q20. See O.P. Gupta's Mathematicia Vol.2 for both the options.

Q21. See O.P. Gupta's Mathematicia Vol.1.

Q22. Radius and height are respectively 4 and 8 (both in cm). **Q23.** $\left(\dfrac{\sqrt{3}+1}{8}-\dfrac{\pi}{6}\right)$ sq. units

Q24. See O.P. Gupta's Mathematicia Vol.1. **OR** Max. profit is ₹76. Amount for charity is ₹7.60.

Q25. See NCERT Exemplar Problems Solutions. **Q26.** $7x+13y+4z-9=0$, $\cos^{-1}\left(\dfrac{4}{3\sqrt{26}}\right)$.