# MATHEMATICS
## CLASS XII

**Time: 3 hours** **MM: 100**

## General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of **26** questions divided into three sections **A, B** and **C**. Section **A** comprises **6** questions of **one mark** each, Section **B** comprises **13** questions of **four marks** each and Section **C** comprises 7 questions of **six marks** each.*
3. *All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.*
4. *There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculator is not permitted. You may ask for logarithmic tables, if required.*

## Section – A

Q1 Order of differential equation is 2 and degree is o. Write true or false with justification.

Q2 Write the integrating factor for solving differential equation

$$2\frac{dx}{dy}+\frac{x}{y}=y^3$$

Q3 $\overrightarrow{AB},\overrightarrow{BC} and \overrightarrow{CA}$ from the sides of an equilateral $\Delta$ of side 2 units. Find $\left|\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}\right|$.

Q4 If A = $\begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix} and \left|A^3\right| = 125, then$ find the value of p.

Q5 A vector $\vec{a}$, makes angles $90^0$, $135^0$, $45^0$ with x, y, and z – axes respectively . If $\left|\vec{a}\right| = 2\sqrt{2}$ units then find the projection of $\vec{a}$, on z – axis.

Q6 Write the name of the coordinate axis to which the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-1}{0}$ is perpendicular.

## SECTION - B

Q7 There are two families P and Q. There are 3 men, 3 women and 12 children in family P and 2 men, 2 women and 4 children in family Q. The recommended daily allowance for calories is :
Man : 2400, woman : 2000, child : 1400 and man : 60 g for proteins is : women : 40 g and child : 35 g.
(i) Represent the above information by matrices. Using matrix multiplication, calculate the total requirement of calories and proteins of each of the two families.
(ii) Which family is an ideal family and why ?

**OR**

The total monthly earnings of a worker in a factory is given by I = p + qx + ry, where X denotes the number of points of honesty and Y denotes the number of points of punctuality . Data for three months are as follows :

| Month | Earning | No. of points X | Y |
|---|---|---|---|
| 1 | 6950 | 40 | 10 |
| 2 | 6725 | 35 | 09 |
| 3 | 7100 | 40 | 12 |

(a) Represent the above data by linear equations and write its matrix form.

(b) Is it possible to solve the system of equations so obtained using matrices?

Which value you prefer to be rewarded most and why ?

Q8 If $x = \operatorname{cosec}\left[\tan^{-1}\left\{\cos\left(\cot^{-1}\left(\sec\left(\sin^{-1}a\right)\right)\right)\right\}\right]$ and,

$y = \sec\left[\cot^{-1}\left\{\sin\left(\tan^{-1}\left(\operatorname{cosec}\left(\cos^{-1}a\right)\right)\right)\right\}\right]$ where $a \in [0,1]$. Find the relationship between x and y in terms of a.

**OR**

If $0 < a < b < c$ prove that $\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)=2\pi$

Q9 Using properties of determinants ,show that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+bc+ca+ab$$

Q10 If $y = \left(\tan^{-1}x\right)^2$, showthat $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right)y_1 = 2$

Q11 If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = O$ Hence find $A^{-1}$.

Q12 Differentiate $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) w.r.t. \sin^{-1}\left(2x\sqrt{1-x^2}\right); \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

OR

Differentiate w.r.t.x.

$$y = \sin^{-1}\left(\frac{2^{x+1}5^x}{1+100^x}\right)$$

Q13 Find $\int \frac{(3\sin\phi-2)\cos\phi}{5-\cos^2\phi-4\sin\phi}\,d\phi$

Q14 Integrate the function: $\frac{(x-3)e^x}{(x-1)^3}$

Q15 Evaluate $\int_{-1}^{\frac{3}{2}} |x\sin(\pi x)| dx$

Q16 Show that the four points with position vectors
$4\hat{i}+8\hat{j}+12\hat{k}, 2\hat{i}+4\hat{j}+6\hat{k}, 3\hat{i}+5\hat{j}+4\hat{k}$ and $5\hat{i}+8\hat{j}+5\hat{k}$ are coplanar.

Q17 Find the foot of perpendicular from the point (2,3,4) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also find the perpendicular distance from the given point to the line.

Q18 Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

OR

In an event, a player has to win all 10 rounds continuously. The probability that he will win each round successfully is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 rounds?

Q19 Find the values of a and b such that $f$(x) defined by

$$f(x)=\begin{cases} \sin\pi x, -1\le x<-\frac{1}{2} \\ ax^2+b, -\frac{1}{2}\le x\le 0 \\ \frac{1}{x}(\operatorname{cosec} x-\cot x), 0<x\le 1 \end{cases}$$

is continuous in [-1, 1].

## SECTION - C

Q20 A , Band C throw a pair die alternatively. A wins the game if he gets a doublet .B wins the game if he gets sum 10 or more than 10. C wins the game if he gets sum 4or less than 4. Find their respective probabilities of winning, if A starts first.

OR

An urn contains 25 balls of which 10 balls bear mark ‘X’ and the remaining 15 bear a mark ‘Y’. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that
(i) all will bear ‘X’ mark.
(ii) not more than 2 will bear ‘Y’ mark.
(iii) at least one ball will bear ‘Y’ mark.
(iv) the number of balls with ‘X’ mark and ‘Y’ mark will be equal.

Q21 A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day. She produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table :

| Items | Number of hours required on machines | | |
|---|---|---|---|
| | I | II | III |
| M | 1 | 2 | 1 |
| N | 2 | 1 | 1.25 |

She makes a profit of Rs 600 and Rs 400 on items M and N respectively. How many of each item should she produce so as to maximize her profit assuming that she can sell all the items that she produced? What will be the maximum profit?

Q22 Find the equation of the plane passing through intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 and whose perpendicular distance from the origin is $\sqrt{2}$ units.

Q23 Show that the given differential equation is homogeneous and solve:

$$\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}ydx=\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}xdy$$

OR

Solve the differential equation : $2ye^{\frac{x}{y}}dx+\left(y-2xe^{\frac{x}{y}}\right)dy=0$ and find its particular solution, given that, x = 0 when y = 1.

Q24 Find the area bounded by the curves: $y^2=4x$, $x=1$, $x-2y+4=0$

Q25 Consider $f:[3,\infty)\rightarrow[-4,\infty)$ given by $f(x) = 3x^2-12x + 5$. Show that $f$ is invertible and find $f^{-1}(y)$

Q26 A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle.

Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

Answers

1 False , Order & degree if defined are positive integers

2 $\sqrt{y}$ 3 0 4 $\pm 3$ 5 2 6 z – axis.

7 Calories proteins

P : 30,000 720g

Q 14,000 340g

Or

(i) p + 40q + 10r = 6950

P + 35q + qr = 6725

P + 40q + 12r = 7100

$$AX = B$$

$$A = \begin{bmatrix} 1 & 40 & 10 \\ 1 & 35 & 9 \\ 1 & 40 & 12 \end{bmatrix}, X = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$B = \begin{bmatrix} 6950 \\ 6725 \\ 7100 \end{bmatrix}$$

B) YES

8 $x^2 + y^2 + 3 - a^2$

11 $$A^2 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix} A^3 = \begin{bmatrix} 63 & 40 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$$A^{-1} = \frac{1}{40}\begin{bmatrix} -4 & 4 & 8 \\ 1 & -11 & 8 \\ 14 & 6 & -8 \end{bmatrix} = \begin{bmatrix} -1/10 & 1/10 & 1/5 \\ 1/40 & -11/40 & 1/5 \\ -1/20 & 3/20 & -1/5 \end{bmatrix}$$

12 $\frac{3\sqrt{1-x^2}}{2\left(1+x^2\right)} or \frac{dy}{dx} = \frac{2.10^x.\log 10}{1+\left(10^2\right)x}$

13 3 log ( $\sin\phi - 2$ ) - $\frac{4}{(\sin\phi - 2)} + C$

14 $\frac{e^x}{(x-1)^2}+C$

15 $\frac{3\pi+1}{\pi^2}$

17 k = $k=\frac{13}{49};\left(\frac{170}{49},\frac{78}{49},\frac{10}{49}\right);D=\frac{1}{49}\sqrt{(72)^2+(69)^2+(86)^2}$

18 Mean = $\frac{34}{221},\operatorname{var}(x)=\frac{6800}{(221)},\sigma=0.37 or 3\times\left(\frac{5}{6}\right)^{10}$

19 a = -6, b = ½

20 $\frac{36}{91},\frac{30}{91},\frac{25}{91}$ or (i) $\left(\frac{2}{5}\right)^6$ (ii) $7\left(\frac{2}{5}\right)^4$ (iii) $1-\left(\frac{2}{5}\right)^6$ (iv) $\frac{864}{3125}$

21 (4,4) profit Rs 4000

22 $\lambda=\frac{-11}{5},-3, eg'sare$ -x + 4y + 9z – 14 = 0 & -x + z – 2 = 0

23 $xy=\sec\left(\frac{y}{x}\right)$ or $2e^{\frac{x}{y}}=y+C/2e^{\frac{x}{y}}=y+2$

24 $\frac{5}{12} sq. onto$ 25 $f^1$ (4) = $2^{+}\sqrt{\frac{y+7}{3}}$