MATHEMATICS

CLASS XII

Code – (3 -16)

Time: 3 hours **MM: 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of **26** questions divided into three sections **A**, **B** and **C**. Section **A** comprises **6** questions of **one mark** each, Section **B** comprises **13** questions of **four marks** each and Section **C** comprises 7 questions of **six marks** each.*
3. *All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.*
4. *There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculator is not permitted. You may ask for logarithmic tables, if required.*

Section A

Q1 Find the order and degree of differential equation $\left(\sqrt{a+x}\right)\frac{dy}{dx}+x=0$ 1

Q2 Find the projection $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$ on the vector $b=4\hat{i}-4\hat{j}+7\hat{k}$ 1

Q3 Find the vector and Cartesian equation of x –axis 1

Q4 If $A=\begin{pmatrix}0 & 1\\ 2 & 4\end{pmatrix}$ and $KA=\begin{pmatrix}0 & 3a\\ 2b & 24\end{pmatrix}$ find K, a ,b 1

Q5 Solve the equation for $x:\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$ 1

Q6 If $y=\sin^{-1}\left(e^{x^2}\right)$ find the $\frac{dy}{dx}$ 1

SECTION B

Q7 Using properties of determinants prove $\begin{vmatrix}a^2 & bc & c^2+ac\\ a^2+ab & b^2 & ac\\ ab & b^2+bc & c^2\end{vmatrix}=4a^2b^2c^2$ 4

Q8 Consider $f:R_+U\{0\}\to[-5,\infty)$ given by f(x) = $9x^2+6x-5$. Show that f is invertible with $f^{-1}(y)=\left(\frac{\sqrt{y+6}-1}{3}\right)$ 4

Q9 Find the equation of the plane which contains the line of intersection of two planes $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$ and $\vec{r}.\left(2\hat{i}+\hat{j}+\hat{k}\right)+5=0$ and which is perpendicular to the plane $\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$ 4

OR

A vector $\vec{n}$ of magnitude 8 units is inclined to x- axis at 45^0, y- axis at $60^{\,0}$ and an acute angle with z axis . If a plane passes through a point $\left(\sqrt{2},-1,1\right)$ and is normal to $\vec{n}$ find the equation of plane in vector form

Q10 Find all the points of discontinuity o f f defined by f(x) = |x| -|x+1| 4

Q11 Using vector find x such that the four points A $\left(3,2,1\right), B\left(4,x,5\right), C\left(4,2,-2\right) and\ D\left(6,5,-1\right)$ are coplanar.

Q12 If $y = x\log\left(\left(a-x\right)^{-1}+a^{-1}\right)$ prove that $x\left(x+1\right)\frac{d^2y}{dx}+x\frac{dy}{dx}=y-1$

OR

Differentiate : $\tan^{-1}\left(\frac{1+ax}{1-ax}\right) w.r.t\sqrt{1+a^2x^2}$

Q13 Evaluate : $\int\frac{\left(3\sin\theta-2\right)\cos\theta}{5-\cos^2\theta-4\sin\theta}d\theta$ 4

OR

$\int\frac{x^2+4}{x^4+x^2+16}dx$

Q14 Evaluate : $\int_0^a \sin^{-1}\sqrt{\frac{x}{a+x}}\ dx$ 4

OR

$\int_0^{\frac{\pi}{2}}\frac{x}{\sin x+\cos x}dx$

Q15 In a game man wins a rupee for a six and loses a rupee for any other number when a fair dice is thrown .The man decided to throw a dice thrice but to quit as and when he gets six. Find the expected value of the amount he wins /loses. 4

Q16 If $y=\left(\sin x\right)^{\tan x}+\left(\cos x\right)^{\sec x}$ find $\frac{dy}{dx}$ 4

Q17 Evaluate as limit of sums $\int_1^3\left(3x^2-5x\right)dx$ 4

Q18 Prove $\tan^{-1}\left(\frac{3\sin 2\alpha}{5+3\cos 2\alpha}\right)+\tan^{-1}\left(\frac{1}{4}\tan\alpha\right)=\alpha$ 4

Q19 Find the coordinates of the points on the curve y = x^2 + 3x + 4, tangents at which pass through the origin . 4

SECTION C

Q20 Using elementary Row transformation fine the inverse of $\begin{pmatrix}-1 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{pmatrix}$ 6

OR

The prices of three food items X, Y, and Z are x, y, z per unit respectively. A purchased one unit of each item and paid Rs. 70. B purchased one unit of X, 2 units of y and 3 units of Z and paid Rs. 160, while C purchased one unit of X, 3 units of Y and 4 units of Z and paid Rs. 220. Use matrices to find the price of each item. If all the three food items are equally good, which item a poor person should purchase.

Q21 Using integration find the area of the region bounded by the curve $\frac{x^2}{9}+\frac{y^2}{4}=1$ and line $\frac{x}{3}+\frac{y}{2}=1$ 6

Q22 The sum of the perimeter of a circle and a square is k. Where k is some constant. Prove that the sum of the areas is least when the side of a square is double the radius of the circle. 6

Q23 Solve the differential equation : 6

$\left(1+y+x^2y\right)dx+\left(x+x^3\right)dy=0$ when y(1) = 0

OR

$x.e^{\frac{y}{x}}-y\sin\frac{y}{x}+x\frac{dy}{dx}\sin\frac{y}{x}=0$ given y(1) = 0

Q24 Two bags A and B contain 4 white and 3 black balls and 2 white and 2 black balls respectively. From bag A, two balls are drawn and then transferred to bag B. A ball is then drawn from bag B and is found to be black ball. What is the probability that the transferred balls were 1 white 1 black? 6

OR

A and B throw pair of dice turn by turn. The first to throw 9 is awarded a prize. If A starts the game show that the probability of a getting the prize is $\frac{9}{17}$. Also find the probability of B winning the game.

Q25 A factory makes tennis rackets and cricket bats. A tennis rackets takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftsman's time . In a day, factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time . If the profit on a racket and a on a bat is Rs. 20 and Rs. 10 respectively , find the number of tennis and cricket bats that the factory must manufacture to earn the maximum profit .Make it as LPP and solve graphically . Why most of the young men play cricket and not tennis? 6

Q26 Show that the line $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$ are coplanar and find equation of plane containing these lines. 6

OR

Find the length and equation of the line of shortest distance between the lines

$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}\ and\ \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$

Also find the points where the line of shortest distance meets the given lines.