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DESIGN OF QUESTION PAPER

CBSE BOARD: 2015-16

Time allowed:3 Hours CLASS:XIISUB:MATHEMATICS(041)*M.Marks:100*

General Instructions: DATE:30/01/2016

i. All questions are compulsory.
ii. The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
iii. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
iv. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

SECTION – A

(one mark each)

1. If $\alpha \le 2\sin^{-1}x + \cos^{-1}x \le \beta$ find the value of α and β
2. If a matrix has 28 elements, how many possible orders it can have? what is it has 13 elements?
3. Write the order of the differential equation of all circles of given radius.
4. Find the integrating factor of the differential equation $\frac{dy}{dx}(x\log x) + y = 2\log x$.
5. Find the angle between $\hat{i} - \hat{j}$ *and* $\hat{j} - \hat{k}$
6. Find the unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ *and* $\hat{i} + \hat{j}$

SECTION – B

(four marks each)

7. Evaluate $\int \frac{2x+3}{\sqrt{3+4x-4x^2}}$ dx.

OR

Evaluate $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$

8. A committee of 4 students is selected at random from a group consisting 8boys and 4 girls. Given that there is at least one girl on the committee, calculate the probability that there are exactly 2 girls on the committee.

OR

A bag contains (2*n* + 1) coins. It is known that *n* of these coins have a head on both sides where as the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of *n*.

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9. Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{i}+2\hat{j}+\hat{k}$ *and* $-\hat{i}+3\hat{j}+4\hat{k}$

10. Show that the straight lines whose direction cosines are given by 2l+2m-n=0 and mn+nl+lm=0 are at right angles.

11. If a_1, a_2, a_3,...,a_n is an arithmetic progression with common difference d, then evaluate the following expression.

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+..............\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]$$

OR

Find the real solutions of the equation $\tan^{-1}\left(\sqrt{x(x+1)}\right)+\sin^{-1}\left(\sqrt{x^2+x+1}\right)=\frac{\pi}{2}$

12. Prove that the function f defined by $f(x)=\begin{cases}\dfrac{x}{|x|+2x^2} & x\neq 0\\ k, & x=0\end{cases}$, remains discontinuous at x=0,regardless the choice of k.

13. If $x^{y}=e^{x-y}$, show that $\frac{dy}{dx}=\frac{\log x}{(\log x+1)^2}$

14. If $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$, show that $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$

15. Evaluate $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

16. Two schools A and B decided to award prizes to their students for three values, team spirit, truthfulness and tolerance at the rate of Rs x, Rs y and Rsz per student respectively. School A, decided to award a total of Rs 1,100 for the three values to 3, 1 and 2 students respectively while school B decided to award Rs 1,400 for the three values to 1, 2 and 3 students respectively. If one prize for all the three values together amount to Rs 600 then
(i) Represent the above situation by a matrix equation after forming linear equations.
(ii) Is it possible to solve the system of equations so obtained using matrices ?
(iii) Which value you prefer to be rewarded most and why ?

17. Let $A=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}$.Then show that A2-4A+7I=0, Using this result calculate A^5 also.

OR

If $A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}$, $B=\begin{bmatrix}2 & 3\\ -1 & 2\end{bmatrix}$ and $x^2=-1$, then show that $(A+B)^2=A^2+B^2$

18. Show that if the determinant $\Delta=\begin{vmatrix}3 & -2 & \sin 3\theta\\ -7 & 8 & \cos 2\theta\\ -11 & 14 & 2\end{vmatrix}=0$,then sinθ=0 or $\frac{1}{2}$

19. Evaluate $I=\int_{-1}^{1}\frac{x^3+|x|+1}{x^2+2|x|+1}dx$

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SECTION – C

(Six marks each)

20. Solve : $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

OR

Solve: $\frac{dy}{dx} = \cos(x + y) + \sin(x + y)$

21. The plane $ax + by = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle α. Prove that the equation of the plane in its new position is $ax + by \pm(a^2 + b^2 \tan\alpha)\, z = 0$.

22. By examining the chest X ray, the probability that TB is detected when a person is actually suffering is 0.99. The probability of an healthy person diagnosed to have TB is 0.001. In a certain city, 1 in 1000 people suffers from TB. A person is selected at random and is diagnosed to have TB. What is the probability that he actually has TB?

23. A man rides his motorcycle at the speed of 50 km/hour. He has to spend Rs 2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to Rs 3 per km. He has atmost Rs 120 to spend on petrol and one hour's time. He wishes to find the maximum distance that he can travel. Express this problem as a linear programming problem. "Speed thrill but kill" Explain?

24. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that the area of triangle is maximum when $\theta = \frac{\pi}{6}$.

OR

Find the points of local maxima, local minima and the points of inflection of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$. Also find the corresponding local maximum and local minimum values.

25. Find the area of the region bounded by the curves $x = at^2$ and $y = 2at$ between the ordinate corresponding to $t = 1$ and $t = 2$.

26. Let A = {1, 2, 3, ... 9} and R be the relation in A ×A defined by (a, b) R (c, d) if $a + d = b + c$ for (a, b), (c, d) in A ×A. Prove that R is an equivalence relation and also obtain the equivalent class [(2, 5)].