CLASS XII

FULL LENGTH

Mathematics **Code : 5/16**

Time: 3 hours **MM: 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of **26** questions divided into three sections **A**, **B** and **C**. Section **A** comprises **6** questions of **one mark** each, Section **B** comprises **13** questions of **four marks** each and Section **C** comprises 7 questions of **six marks** each.*
3. *All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.*
4. *There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculator is not permitted. You may ask for logarithmic tables, if required.*

Section - A

Q1 The value of a determinant of order three is 11. Find the value of the square of the determinant formed by replacing the elements with their co-factors of this determinant.

Q2 If θ is an acute angle and the vector $(\sin\theta)\,\hat{i}+(\cos\theta)\,\hat{j}$ is perpendicular to the vector $\hat{i}-\sqrt{3}\hat{j}$, then find θ.

Q3 Cartesian equations of a line are $3x+1=6y-2=1-z$. Find the vector equation of this line.

Q4 Find the angles at which the following vector is inclined to each of the coordinate axes.
$\vec{a}=\hat{i}-\hat{k}$

Q5 Write the Integrating factor for solving differential equation
$$\cos x\frac{dy}{dx}+2y=\sec x+\tan x$$

Q6 If f is a function defined on a closed interval [a,b]. write the condition for continuity of f at a .

	SECTION - B
Q7	The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.
Q8	Express $\tan^{-1}\frac{\cos x}{1-\sin x}, \frac{-3\pi}{2}<x<\frac{\pi}{2}$ in the simplest form.
Q9	If x, y, z are different and $\Delta=\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}=0$, then show that 1 + xyz = 0

Q10	A teacher prepared a performance grade criteria for +2 students on the basis of the numbers of x hours devoted by the students. $f(x)=\begin{cases}1, & \text{if } x\le 3\\ ax+b, & \text{if } 3<x<5\\ 7, & \text{if } x\ge 5\end{cases}$ $\Rightarrow\begin{cases}\text{Grade 1, unsatisfactory x} \le 3\\ \text{Grade (ax + b),satisfactory x = 4}\\ \text{Grade 7, Average x} \ge 5\end{cases}$ Find the value of a and b , so that the function f given above is continuous at x = 3 and x = 5. Also, find the marking grade for 'satisfactory'. **What analysis can be drawn from above grade card prepared by the teacher?**
Q11	Prove that the function f given by $f(x)=\lvert x+2\rvert, x\in R$ in continuous at x = -2 but is not differentiable at x = - 2.
Q12	Differentiate the function : $\left(x+\frac{1}{x}\right)^x+x^{\left(1+\frac{1}{x}\right)}$ **OR** Differentiate the function : $x^{x\cos x}+\frac{x^2+1}{x^2-1}$
Q13	Integrate the function : $\frac{1}{\sqrt{8+3x-x^2}}$
Q14	Find $\int\frac{(3\sin\phi-2)\cos\phi}{5-\cos^2\phi-4\sin\phi}d\phi$.
Q15	Evaluate $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\sin^2 x dx$ **OR** Evaluate the integrals : $\int_0^{\pi}\frac{xdx}{1+\sin x}$
Q16	If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i},\hat{j}$ and $\hat{k},\vec{\alpha}=3\hat{i}-\hat{j},\vec{\beta}=2\hat{i}+\hat{j}-3\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta}=\vec{\beta}_1+\vec{\beta}_2$ ' where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.
Q17	Find the values of p so that the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. **OR** Find the shortest distance between the lines whose vector equations are $\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ and $\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$
Q18	In a hostel, 60% of the students read Hindi newspaper, 40% read English news paper and 20% read both Hindi and English newspapers. A student is selected at random. (a) Find the probability that she reads neither Hindi nor English newspapers. (b) If she reads Hindi news paper, find the probability that she reads English newspaper. (c) If she reads English news paper, find the probability that she reads Hindi news paper.

	OR An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red ?
Q19	Two shopkeepers A and B of a particular school have stocks of books on Moral Education, Non – violence and Truth as given by the matrix $M.Edu.$ N- Violence Truth $\begin{bmatrix} 36 & 48 & 24 \\ 24 & 12 & 60 \end{bmatrix} \begin{matrix} ShopA \\ ShopB \end{matrix}$ If the selling prices of these books are respectively Rs. 300, Rs 200 Rs. 250 per book respectively, find the total amount received by each shopkeeper, if all the books are sold, using matrices. Which book preferred the most and why?
	SECTION - C
Q20	A manufacturer has three machine operators A,B and C. The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?
Q21	If a young man rides his motorcycle at 25km/hour, he had to spend Rs 2 per km on petrol. If he rides at a faster speed of 40 km/ hour, the petrol cost increase at Rs 5 per km. He has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this as LPP and solve it graphically. **Which mode of transport you suggest to a young man and why.**
Q22	Find the distance of the point (-1, -5, -10) from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ *and* the plane $\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 5.$
Q23	Solve the differential equation $\left(x \text{ dy - y dx}\right) y \sin\left(\frac{y}{x}\right) = \left(ydx + xdy\right) x \cos\left(\frac{y}{x}\right).$ OR For the following differential equation , find the particular solution satisfying the given condition $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x \left(x \neq 0\right); y = 0$ when $x = \frac{\pi}{2}$
Q24	Find the area of the region in the first quadrant enclosed by the x- axis, the line y = x, and the circle $x^2 + y^2 = 32.$ OR The area between $x = y^2$ and x = 4 is divided into two equal parts by the line x = a, find the value of a.
Q25	Let * be the binary operation on **N** given by a*b = L.C.M . of a and b . Find (i) 5*7, 20 * 16 (ii) Is * commutative ? (iii) Is * associative ? (iv) Find the identity of * in **N** (v) Which elements of **N** are invertible for the operation * ?
Q26	If lengths of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum.

ANSWERS

1 14641 **2** $\theta = \frac{\pi}{3}$ **3** $\vec{r} = (\frac{-1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k}) + \lambda\left(2\hat{i} + \hat{j} - 6\hat{k}\right)$ **4** $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right), \frac{\pi}{2}, \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

5. $(\sec x + \tan x)^2$ **6.** $\lim_{x \to a^+} f(x) = f(a)$ **7** x = 1, y = 2, z = 3

8 $\frac{\pi}{4} + \frac{x}{2}$ **10** Grade 4, (more study hours better the results.)

12 $\left(x + \frac{1}{x}\right)^x \left[\frac{x^2 - 1}{x^2 + 1} + \log\left(x + \frac{1}{x}\right)\right] + x^{1+\frac{1}{x}}\left(\frac{x + 1 - \log x}{x^2}\right)$

Or $x^{x\cos x}\left[\cos x.(1 + \log x) - x\sin x \log x\right] - \frac{4x}{\left(x^2 - 1\right)^2}$

13 $\sin^{-1}\left(\frac{2x - 3}{\sqrt{41}}\right) + C$ **14** $3\log(2 - \sin\phi) + \frac{4}{2 - \sin\phi} + C$ ($\sin ce, 2 - \sin\phi$ is always positive)

19 Amount received by shopkeeper A = Rs. 27600

16 $\vec{\beta} = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j} and \vec{\beta}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$ **17** $p = \frac{70}{11}$ **or** $\frac{8}{\sqrt{29}}$

18 (a) $\frac{1}{5}, (b)\frac{1}{3}, (c)\frac{1}{2}$ **or** $\frac{1}{2}$ **20** $\frac{5}{34}$ **21** d is maximum, when young man travel $\frac{50}{3}$ km at a speed of 25km/hour and $\frac{40}{3}$ km at a speed of 40 km/ hour. Maximum distance = 30km **22** 13

23 $\sec\left(\frac{y}{x}\right) = C$ xy OR $y\sin x = 2x^2 - \frac{\pi^2}{2}(\sin x \neq 0)$ **24** 4π OR $(4)^{\frac{2}{3}}$

25 (i) 5* 7 = 35, 20*16 = 80 (ii) yes (iii) yes (iv) 1 (v) 1 **26** $75\sqrt{3} cm^2$