School Of Math

MATHEMATICS

CLASS XII

CODE: E/I/3/16

Time: 3 hours **MM: 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of **26** questions divided into three sections **A**, **B** and **C**. Section **A** comprises **6** questions of **one mark** each, Section **B** comprises **13** questions of **four marks** each and Section **C** comprises 7 questions of **six marks** each.*
3. *All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.*
4. *There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculator is not permitted. You may ask for logarithmic tables, if required.*

SECTION – A

Q1 Find the integrating factor of differential equation $\frac{dy}{dx} = e^{\frac{x^2}{2}} + xy$. 1

Q2 If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} = P + Q, where\ P$ is symmetric matrix and Q is skew – symmetric matrix , then find the matrix P. 1

Q3 Write the degree of differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 6y^5 = 0$. 1

Q4 Find the angle between vectors $\vec{a} and \vec{b}$ such that $|\vec{a}| = \frac{2}{\sqrt{7}} and |\vec{b}| = \sqrt{7}$ $|\vec{a} \times \vec{b}| = 1$. 1

Q5 Find the equation of line parallel to X – axis and passing through the origin. 1

Q6 Find the value of $\hat{i}.\left(\hat{j} \times \hat{k}\right) - \hat{j}.\left(\hat{k} \times \hat{i}\right) + \hat{k}.\left(\hat{i} \times \hat{j}\right)$. 1

SECTION – B

Q7 If two matrices are $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, then verify that BA = 6*I* and use the result to solve the following system of linear equations 4

x – y = 3

2x + 3y + 4z = 17

and y + 2z = 7.

OR

Find the inverse of $A=\begin{bmatrix}3 & 0 & -1\\ 2 & 3 & 0\\ 0 & 4 & 1\end{bmatrix}$, by elementary row transformations.

Q8 Find the value of $\tan\left[\frac{1}{2}\left\{\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}\right], |x|<1, y>0, xy<1.$ 4

Q9 Using properties of determinants , prove that $\begin{vmatrix}b+c & c+a & a+b\\ q+r & r+p & p+q\\ y+z & z+x & x+y\end{vmatrix}=2\begin{vmatrix}a & b & c\\ p & q & r\\ x & y & z\end{vmatrix}$. 4

or

Find the equation of the line joining the points A(1,3) and B(0,0) . By using determinants, find the value of a, if C (a,0) is a point such that the area of ΔABC is 3 sq units.

Q10 Evaluate $\int \frac{1}{x\left(x^3+8\right)}dx.$. 4

Q11 Verify the hypothesis and conclusion of Lagrange's mean – value theorem for the function 4

f(x) = $\frac{1}{4X-1}, 1\le x\le 4.$

OR

Verify Rolle's theorem for the function

$f(x)=\log\left(\frac{x^2+ab}{(a+b)x}\right)$ in [a,b], where 0<a<b.

Q12 Discuss the continuity of the function 4

$$f(x)=\begin{cases}\frac{3}{2}-x, & if \quad \frac{1}{2}\le x<1\\ \frac{3}{2}, & if \quad x=1\\ \frac{3}{2}+x, & if \quad 1<x\le 2\end{cases}$$ at x = 1 .

Q13 Evaluate: $\int_{-1}^{2}(7x-5)\,dx$, as a limit of sum. 4

Q14 Evaluate: $\int_3^4 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}}dx$. 4

OR

Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$.

Q15 A company has two plants for manufacturing scooters. Plant I manufactures 70% of the scooters and Plant II manufactures 30% of the scooters. At plant I, 30% of the scooters are maintaining pollution norms and at plant II, 90% of the scooters are maintaining pollutions norms. A scooter is chosen at random and is found to be fit on pollution norms. Find the probability that it has come from plant II. What is the importance of pollution norms for a vehicle? 4

Q16 Find a vector of magnitude 4 units perpendicular to each of vectors 4
$\left(\vec{a}+\vec{b}\right)$ *and* $\left(\vec{a}-\vec{b}\right)$, *where* $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ *and* $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}.$

Q17 Find the coordinates of the point on line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{6}$, which are at a distance of 3 units from the point (1, -2, 3). 4

Q18 Show that $A=\begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$ satisfies the equation $A^2 - 3A - 7I = 0$ and hence find the value of A^{-1}. 4

Q19 In a kite festival, a kite is at a height 120m and 130m string is out. If the kite is moving horizontally at the rate of 5.2m/s. then find the rate at which string is pulled out at that instant. 4

SECTION – C

Q20 Let A = {1,2,3,…,9} and R be the relation in $A\times A$ defined by (a,b) R (c,d), if a + d = b + c for a,b,c,d $\in A$. Prove that R is an equivalence relation. Also , obtain the equivalence class [(2,5)]. 6

OR

Let f: $N\rightarrow R$ be a function defined as f(x) = $4x^2 + 12x + 15$. Show that f: $N\rightarrow S$ is invertible, where S is the range of f. Also, find inverse of f.

Q21 Solve the differential equation $\left(x^2-y^2\right)dx+2xydy=0$, given that y = 1 when x = 1. 6

Q22 Find the equation of plane passing through the points (1,2,3),(0,-1,0) and parallel to the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z}{-3}$. 6

OR

Find the equation of the plane passing through the line of intersection of planes 2x + y – z = 3 , 5x – 3y + 4z + 9 = 0 and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.

Q23 A house wife wishes to mix up two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. 6
The vitamin contents of 1 kg of food X and 1 kg of food Y are as given in the following table

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

If 1 kg of food X costs Rs 6 and 1 kg of food Y costs Rs 10, then find the least cost of the mixture which will produce the desired diet.

Q24 A jet of enemy is flying along the curve y = x^2 + 2 and a soldier is placed at the point (3,2). Find the minimum distance between the soldier and the jet. 6

Q25 Using integration find the area of the triangle formed by x- axis and tangent and normal to the curve y^2 = 4x at (4,4). 6

Q26 A bag contains 25 balls of which 10 are purple and the remaining are pink. A ball is drawn at random, its colour is noted and it is replaced. 6 balls are drawn in this way. Find the probability : 6
(i) All balls were purple. (ii) Not more than 2 were pink.
(iii) An equal number of purple and pink balls were drawn.
(iv) At least one ball was pink.