M.M: 100 **Revision Paper: Mathematics** **Time: 3Hrs.**

Important Instructions:

1. **All questions are compusary.**
2. **Question number 1 to 6 carry 1 mark each,question number 7 to 19 carry 4 marks each, Question number 20 to 26 carry 6 marks each.**
3. **There is no overall choice. However, internal choices are provided in few Questions. You have to attempt one such choice.**

SECTION A

1. Consider the binary operation $\vee$ on the set {1, 2, 3, 4, 5} given by $a \vee b = \min\{a, b\}$.Write the value of $(2 \vee 3) \vee 5$.
2. Find the values of each of the following: $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$
3. Compute the indicated products. $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$.
4. Integrate the function: $\dfrac{x^3 \sin\left(\tan^{-1}x^4\right)}{x^8+1}$.
5. If | A | = 2, where A is 2 × 2 matrix, find | adj A |.
6. Find the equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane.

SECTION B

7. Show that the relation R in the set **R** of real numbers, defined as R = $\left\{(a,b): a \le b^2\right\}$ is neither reflexive nor symmetric nor transitive.

OR

Given a non-empty set X, consider the binary operation : P(X) × P(X) → P(X) given by A * B = A ∩ B ∀A, B in P(X), where P(X) is the power set of X. Show that X is the identity element for this operation and X is the only invertible element in P(X) with respect to the operation *.

8. Prove **that:** $\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \dfrac{x}{2}, x \in \left(0, \dfrac{\pi}{4}\right)$

9. Using the property of determinants and without expanding prove that:

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

10. Let

$$f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & , \text{ if } x<0 \\ a & , \text{ if } x=0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & , \text{ if } x>0 \end{cases}$$

Determine the value of a so that f(x) is continuous at x = 0.

11. Find **dy/dx for the functions** $(\cos x)^y = (\cos y)^x$.

12. Sand is pouring from a pipe at the rate of 12 cm3/s. The falling sand forms a cube on the ground in such a way that the height of the cone is always one-sixth of radius of the base. How fast is the height of the sand cone increasing when height is 4 cm?

OR

Prove that $y = \dfrac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function of θ in $\left[0, \dfrac{\pi}{2}\right]$.

13. Evaluate $\displaystyle\int_{-1}^{3/2} |x \sin x| dx$

14. Form the D.E of the family of curves by eliminating the arbitrary constants. :
$y = e^{2x}(a+bx)$.

15. Evaluate $\int (2x+3)\sqrt{x^2+4x+3}\, dx$.

16. Find a particular solution for the D.E. satisfying the given condition: $\left(1+x^2\right)\dfrac{dy}{dx} + 2xy = \dfrac{1}{1+x^2}$; $y=0$ When x=1.

17. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.

OR

Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is π/6, prove that $[\vec{a}\ \vec{b}\ \vec{c}]^2 = \frac{1}{4}|\vec{a}|^2|\vec{b}|^2$.

18. . Find the shortest distance between the lines whose vector equations $\vec{r} = (1-t)\vec{i} + (t-2)\vec{j} + (3-2t)\vec{k}$ and $\vec{r} = (s+1)\vec{i} + (2s-1)\vec{j} + (2s+1)\vec{k}$.

19. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(i) the problem is solved (ii) exactly one of them solves the problem.

OR

Three persons A, B, C throw a die in succession till one gets a 'six' and wins the game. Find their respective probabilities of winning, if A begins.

SECTIONC

20. Find the inverse the matrix by using elementary transformation: $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

21. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

OR

An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

22. Evaluate the definite integrals: $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x}\,dx$

23. Find the area of the region $\{(x, y) : y^2 \leq 4x,\ 4x^2 + 4y^2 \leq 9\}$.

24. Find the vector equation of the line passing through the point (1, 2, - 4) and perpendicular to the two lines:

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

OR

Find the equation of the plane passing through the point A(1, 2, 1) and perpendicular to the line joining the points P(1, 4, 2) and Q(2, 3, 5). Also, find the distance of this plane from the line $\frac{x+3}{2}=\frac{y-5}{-1}=\frac{z-7}{-1}$.

25. One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and l kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.
26. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, 'what is the probability that a person has the disease given that the test result is positive?

By: Sumit Luthra. PGT maths, Mount Carmel School ,Dwarka,New Delhi.

Email: sumit.luthra1981@gmail.com

Solutions are available on: 100math.weebly.com