

CLASS XII

GUESS PAPER

MATHEMATICS

Duration: 3 hr.

M.M: 100

General Instructions:

- (i) All questions are compulsory.
- (ii) This question paper contains **29** questions.
- (iii) Question **1–4** in **Section A** are very short-answer type questions carrying **1** mark each.
- (iv) Question **5–12** in **Section B** are short-answer type questions carrying **2** marks each.
- (v) Question **13–23** in **Section C** are long-answer-I type questions carrying **4** marks each.
- (vi) Question **24–29** in **Section D** are long-answer-II type questions carrying **6** marks each.
- (vii) Use of calculators is not permitted.

SECTION – A

1. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i} - 3\hat{j} + 6\hat{k}$.
2. If A is a square matrix such that $|A| = 5$. Write the value of $|AA^t|$.
3. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in \mathbb{R}$, then find $f \circ g(7)$.
4. Find the integrating factor of the differential equation: $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \right) dx = 1 \cdot dy$.

SECTION - B

5. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$. Find a matrix D such that $CD - AB = 0$.
6. If $y = x^{x^{x^{\dots}}}$, then find that $x \frac{dy}{dx} = \frac{y^2}{1 - y \log x}$
7. Sand is pouring from a pipe at the rate of $12 \text{ cm}^2/\text{sec}$. The falling sand form a cone on the ground in such a way that the height of the curve is always one-sixth of the radius of the base. How fast in the height of the sand cone increasing when the height is 4cm?
8. Find the approximate change in the volume V of a cube of side 'x' metres caused by increasing the side by 2%.
9. Evaluate: $\int \frac{\sqrt{9 - (\log x)^2}}{x} dx$.
10. Form the differential equation of the family of ellipses having foci on x-axis and centre at the origin.
11. If the sum of two unit vectors is a unit vector, show that magnitude of their difference is $\sqrt{3}$.
12. A die is thrown twice and the sum of the numbers appearing is observed to be 7. What is the conditional probability that the number 2 has appeared atleast once?

SECTION - C

13. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1 + 3 \cos^2 x} dx$

OR

Evaluate: $\int_0^1 \cot^{-1}(1 - x + x^2) dx$

14. Find the shortest distance between the following lines:

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k} \text{ and } \vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} + (2s+1)\hat{k}$$

OR

Find the equation of the plane passing through the line of intersection of the planes $2x + y - z = 3$ and $5x - 3y + 4z + 9 = 0$ and is parallel to the line

$$\frac{x-1}{2} = \frac{y-3}{4} = \frac{5-z}{-5}.$$

15. Evaluate: $\int \frac{\sin x - x \cos x}{x(x + \sin x)} dx$

16. Prove that $2 \sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$.

OR

Solve the equation for x: $\cos(\tan^{-1}x) = \sin(\cot^{-1}3/4)$.

17. For what value of k is the following function continuous at $x = \frac{-\pi}{6}$?

$$f(x) = \begin{cases} \frac{\sqrt{3} \sin x + \cos x}{x + \frac{\pi}{6}}, & x \neq \frac{-\pi}{6} \\ k, & x = \frac{-\pi}{6} \end{cases}$$

18. The monthly income of Aryan and Babban are in the ratio 3:4 and their monthly expenditure are in the ratio 5:7. If each saves Rs 15000 per

month, find their monthly incomes using matrix method. Which value is reflected in this problem?

19. Differentiate $\tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$ w.r.t. $\cos^{-1} x^2$.

20. Find the general solution of the differential equation:
 $(1 + \tan y) (dx - dy) + 2x dy = 0$.

OR

Solve the following differential equation: $\left(1 + e^{\frac{x}{y}} \right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y} \right) dy = 0$.

21. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

22. In a certain college 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of that students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 metres. Find the probability that the selected student is a girl.

23. Prove that if E and F are independent events, then so are the events E and F'.

SECTION - D

24. Find the equation of the plane which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and whose x - intercept is twice its z - intercept.

Hence write the vector equation of a plane passing through the point (2, 3, -1) and parallel to the plane obtained above.

25. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\sin^{-1} \sqrt{\frac{2}{3}}$.

OR

If the function $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1$, where $m > 0$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then find the value of m .

26. On the set $\{0, 1, 2, 3, 4, 5, 6\}$ a binary operation $*$ is defined as:

$$a * b = \begin{cases} a + b, & \text{if } a + b < 7 \\ a + b - 7, & \text{if } a + b \geq 7 \end{cases}$$

Write the operation table of the operation $*$ and prove that zero is the identity for this operation and each element $a \neq 0$ of the set is invertible with $7 - a$ being the inverse of a .

27. Using property of determinants, prove that

$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

OR

If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and $A^3 - 6A^2 + 7A + kI_3 = 0$ find k .

28. A company manufactures two types of cardigans: type A and type B. It costs Rs 360 to make a type A cardigan and Rs 120 to make type B cardigan. The company can make at most 300 cardigans and spend at most Rs 72000 a day. The number of cardigans of type B cannot exceed

the number of cardigans of type A by more than 200. The company makes a profit of Rs 100 for each cardigan of type A and Rs 50 for every cardigan of type B. Formulate this problem as a linear programming problem to maximize the profit of a company. Solve it graphically and find maximum profit.

29. Using integration, find the area of the region bounded by the curves $y = \sqrt{4 - x^2}$, $x^2 + y^2 - 4x = 0$ and the x-axis.

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