CLASS XII
GUESS PAPER
MATHS

Section – A (10 X 1 = 10 Marks)

1. If $4\cos^{-1}x + \sin^{-1}x = \pi$, then write the value of x.
2. If a function f:R→R is defined as f(x) = $\begin{cases} x+2, & 1 \le x \le 3 \\ 4x-5, & 3 < x < 5 \end{cases}$, find x, when f(x) = 13.
3. If A = $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$, then find A^4
4. If A = $\begin{pmatrix} 0 & 2 \\ 3 & -4 \end{pmatrix}$, $kA = \begin{pmatrix} 0 & 3a \\ 2b & 24 \end{pmatrix}$ then find k, a and b.
5. For two matrices A and B, If AB = A and BA = B, then what is B^2?
6. Find the angle θ, 0<θ<π/2, which increases twice as fast as its sine.
7. If y = $\sin^{-1}3x + \cos^{-1}3x$, x∈[–1/3, 1/3], then what is the value of dy/dx.
8. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x-axis at angles 30^o and 120^o respectively, then what is the value of $|\vec{a}+\vec{b}|$?
9. If $\vec{a}$ and $\vec{b}$ are parallel vectors and $\vec{p} = \vec{a}+\vec{b}$, $\vec{q} = \vec{a}-\vec{b}$, then what is the value of $\vec{p} x \vec{q}$?
10. Write the equation of the plane through (2,3,4) and parallel to the plane x–2y+4z=5.

Section – B (12 X 4 = 48 Marks)

11. Prove that $\tan^{-1}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$, $\frac{-1}{\sqrt{2}} \le x \le 1$

(OR)

Solve for x if $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$.

12. Express the matrix B = $\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

13. Let $f(x)\begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x} & \text{for } x<\dfrac{\pi}{2} \\ a & \text{for } x=\dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & \text{for } x>\dfrac{\pi}{2} \end{cases}$ be continuous at $x = \pi/2$, then find a and b.

14. Compute the area bounded by $y=\dfrac{3x^2}{4}$ and $3x-2y+12=0$.

15. Evaluate $\displaystyle\int_4^9 \frac{\sqrt{x}}{\left(30-x^{\frac{2}{3}}\right)^2}dx$

16. Evaluate $\displaystyle\int_2^7 \left(|x-2|+|x-4|+|x-7|\right)dx$

17. Form the differential equations of the family of curves $xy=Ae^x+Be^{-x}+x^2$, where A and B are arbitrary constants.

18. Solve the differential equation $2y.e^{\frac{x}{y}}dx+(y-2xe^{\frac{x}{y}})dy=0$, given that when x = 0, y = 1.

(OR)

Solve $\cos^3 x\dfrac{dy}{dx}-y\sin x\cot x=\cos x$, given y = 1, when $x = \pi/4$.

19. Show that if $f:R-\left\{\dfrac{7}{5}\right\}\to R-\left\{\dfrac{3}{5}\right\}$ is defined by $f(x)=\dfrac{3x+4}{5x-7}$ and $g:R-\left\{\dfrac{3}{5}\right\}\to R-\left\{\dfrac{7}{5}\right\}$ is defined by $f(x)=\dfrac{7x+4}{5x-3}$ then fog = I_A and gof = I_B, where. $A=R-\left\{\dfrac{3}{5}\right\}$, $B=R-\left\{\dfrac{7}{5}\right\}$, $I_A(x) = x$ on A , $I_B(x) = x$ on B are called identity functions on sets A and B respectively.

20. If $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$ .Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}\cdot\vec{d}=15$.

21. A variable plane is at a constant distance 3p from the origin and meets the axes in A,B,C respectively, then show that locus of the centroid of triangle ABC is $\dfrac{1}{x^2}+\dfrac{1}{y^2}+\dfrac{1}{z^2}=\dfrac{1}{p^2}$.

22. In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Section – C (7 X 6 = 42 Marks)

23. If $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = O$, Hence find A^{-1}.

(OR)

Using elementary transformations, find the inverse of matrix $A = \begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$

24. A wire of length 28cm is cut into two parts which are bent respectively in the form of a square and a circle. Find the length of the wire cut so that the combined area is minimum.

(OR)

Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$

25. Using integration, find the area of the region bounded by the lines $2x+y = 4$, $3x-2y = 6$ and $x-3y+5 = 0$.

(OR)

Find the area lying above the x-axis and included between the circle $x^2+y^2=8x$ and the parabola $y^2=4x$

26. Evaluate $\int e^{\sin x}\left(\frac{x\cos^3 x - \sin x}{\cos^2 x}\right)dx$

27. From the point P(1,2,4) a perpendicular is drawn on the plane $2x+y-2z+3 = 0$. Find the equation, the length and co-ordinates of the foot of perpendicular. Also find the image of P in the plane.

28. Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

29. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll

respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?

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