CLASS XII
GUESS PAPER
MATHEMATICS

Time:3hrs **Maximum Marks: 100**

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections – A, B & C Section A contains 10 questions of 1 marks each. Section B contains12 questions of 4 marks each. Section C contains 7 questions of 6 marks each.

(iii) Use of calculators is not permitted.

SECTION – A

Q.1 If $f(x) = \frac{x}{\sqrt{1+x^2}}$, find (fofof)(x) {**Ans:** $\frac{x}{\sqrt{1+3x^2}}$ }

Q.2 If $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$ then what is the condition on x & y. {**Ans:** xy < 1}

Q.3 If B is a non- singular matrix and A is a square matrix such that $B^{-1}AB$ exits, then what will be the det($B^{-1}AB$) {**Ans:** det(A)}

Q.4 A = [a_{ij}] is a 2x2 matrix whose elements are of type a_{ij} = i – j, name which type of matrix is it. {**Ans:** skew symmetric}

Q.5 If A is a square matrix of 3 x 3 order and $|A| = 5$, find the value of $|A\ adjA|$ {**Ans:** 125}

Q.6 Find a unit vector in direction of $\vec{a} - \vec{b}$ where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ & $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ {**Ans:** $\frac{-3\hat{i} + 4\hat{k}}{5}$ }

Q.7 Write a unit vector in XY- plane, making an angle of 30^o with the positive direction of x-axis.

{**Ans:** $\vec{r} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$**}**

Q.8 Find the vector equation of plane which is at distance of 6 units from the origin and which is normal to the vector $2\hat{i} - \hat{j} + 2\hat{k}$. **{Ans:** $\vec{r}.\left(\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right) = 6$

Q.9 If f'(x) = $\sqrt{x}$, f(1) = 2, find f(x). {**Ans:** $\frac{2}{3}x\sqrt{x} + \frac{4}{3}, x \geq 0$ }

Q.10 Find the maximum and minimum values if any for f(x) = $|x|$, $x \in R$.

{**Ans:** minimum value of 'f' is 0 and 'f' has no maximum value}

SECTION – B

Q.11 If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$

OR

Solve the equation: $\tan^{-1}\sqrt{x^2+x} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ {**Ans:** 0,-1}

Q.12 If R1 and R2 are equivalence relation in a set A, show that $R1 \cap R2$ is also an equivalence relation.

Q.13 Using the properties of determinants, prove that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

Q.14 Prove that $f(x) = \begin{cases} |x|; & x \le 2 \\ [x], & x > 2 \end{cases}$ is continuous on [0,2].

OR

If $y = b\tan^{-1}\left(\frac{x}{a} + \tan^{-1}\frac{y}{x}\right)$, find $\frac{dy}{dx}$ **{Ans:** $\frac{\frac{b}{a} - \frac{by}{x^2+y^2}}{1+\left(\frac{x}{a}+\tan^{-1}\frac{y}{x}\right)^2 - \frac{bx}{x^2+y^2}}$ **}**

Q.15 Show that the line $\vec{r} = (2\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ is parallel to the plane $\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$. Also, find the distance between them. **{Ans:** $\frac{10}{3\sqrt{3}}$ **units.}**

Q.16 Solve the differential equation: $x\frac{dy}{dx} + y - x + xy\cot x = 0 (x \neq 0).$

{Ans: $y = -\cot x + \frac{1}{x} + c(x\sin x)^{-1}.$ **}**

Q.17 Solve the differential equation: $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$

{Ans: $\sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| + \sqrt{1+y^2} = c$ **}**

Q.18 If the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ intersect orthogonally, prove that: $\frac{1}{a} - \frac{1}{a'} = \frac{1}{b} - \frac{1}{b'}$

OR

Find the interval where the function 'f' given by f(x) = sinx + cosx , $0 < x < 2\pi$ is strictly increasing or strictly decreasing. **{Ans:** strictly increasing in $\left(0,\frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$, strictly decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ **}**

Q.19 A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die. **{Ans:** $\frac{625}{23328}$ **}**

Q.20 Integrate $\int \frac{\sin x + \cos x}{\cos^2 x + \sin^4 x}\,dx$ **{Ans:** $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\sin x-\cos x}{\sqrt{3}-\sin x+\cos x}\right| + \tan^{-1}(\sin x - \cos x) + c$ **}**

OR

Integrate $\int \frac{x+\sin x}{1+\cos x}\,dx$ **{Ans:** $x\tan\frac{x}{2} + c$ **}**

Q.21 For any vectors $\vec{a}$ and $\vec{b}$ prove that : $\left(1+|\vec{a}|^2\right)\left(1+|\vec{b}|^2\right) = \left\{(1-\vec{a}.\vec{b})\right\}^2 + \left|\vec{a}+\vec{b}+(\vec{a}\times\vec{b})\right|^2$

Q.22 Given $\cos\frac{x}{2}.\cos\frac{x}{4}.\cos\frac{x}{8}.....\infty = \frac{\sin x}{x}$, prove $\frac{1}{2^2}\sec^2\frac{x}{2} + \frac{1}{2^4}\sec^2\frac{x}{4} + ... = \text{cosec}^2 x - \frac{1}{x^2}$

SECTION – C

Q.23 Given that A = $\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ verify that : $(\text{Adj.A})^{-1} = \text{Adj.}(A^{-1})$

OR

Using elementary transformation find the inverse of matrix: $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Q.24 Using integration find the area of the region bounded by the curve y = x | x |, x-axis and the ordinates x = -1 and x = 1. **{Ans: 2/3 sq. units}**

Q.25 A manufacturer has three machines I, II, III installed in his factory. Machines I and II are capable of being operated for at most 12 hours where as machine III must be operated for at least 5 hours a day. She

produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing I unit of each of M and N on the three machines are given as :

Items	No. of hrs required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs 600 and Rs 400 on items M and N resp. How many of each item should she produce so as to maximize her profit assuming that she can sell all the items that she produced? What will be the maximum profit?

Q.26 Find the equation of plane passing through the point (1,1,1) and containing the line $\vec{r} = \left(-3\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 5\hat{k}\right)$. Also show that the plane contains the line $\vec{r} = \left(-\hat{i} + 2\hat{j} + 5\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} + 5\hat{k}\right)$

Q.27 Evaluate $\int_{-1}^{1/2} |x\cos(\pi x)|dx$ **{Ans:** $\frac{3}{2\pi} - \frac{1}{\pi^2}$ **}**

Q.28 A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$.

OR

An open toped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

Q.29 A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as the other two operators B and C produces 5% and 7% defective items resp. A is on job

for 50% of the time, B on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

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