

# CLASS XII GUESS PAPER-01 MATHS

Time: 3Hours

Max. Marks: 100

### Section-A

[1×10 =10]

1. Find the number of all binary operations on the set { a, b}
2. Find the value of  $\cos(\sec^{-1}x + \operatorname{cosec}^{-1}x)$ ,  $|x| \geq 1$
3. Show that the points A (a, b + c), B (b, c + a), C (c, a + b) are collinear.
4. Find  $P^{-1}$ , if it exists, given that  $P = \begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$
5. Find the point on the curve  $y = x^3 - 11x + 5$  at which the tangent is  $y = x - 11$ .
6. Write the direction ratio's of the vector  $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$  and hence calculate its direction cosines.
7. Find the angle between the two planes  $3x - 6y + 2z = 7$  and  $2x + 2y - 2z = 5$ .
8. Evaluate:  $\int \tan x \tan 2x \tan 3x \, dx$
9. Find  $|\vec{a}|$  and  $|\vec{b}|$ , if  $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$  and  $|\vec{a}| = 8|\vec{b}|$ .
10. If  $f(x) = x^2 - 5x + 7$  and  $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ , find  $f(A)$ .

### Section-B

[4×12 =48]

11. If  $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$ , then find the value of x.
12. Let  $f: \mathbb{N} \rightarrow \mathbb{N}$  be defined by  $f(n) = \begin{cases} n - 1, & \text{if } n \text{ is odd} \\ n + 1, & \text{if } n \text{ is even} \end{cases}$  Show that  $f$  is a bijection.

OR

Let  $\mathbb{N}$  be set of all natural numbers and let  $R$  be a relation on  $\mathbb{N} \times \mathbb{N}$ , defined by  $(a, b) R (c, d) \Leftrightarrow ad = bc$ . Show that  $R$  is an equivalence relation.

13. Using properties of determinants, prove that  $\begin{vmatrix} b^2 + c^2 & ab & ac \\ ba & c^2 + a^2 & bc \\ ca & cb & a^2 + b^2 \end{vmatrix} = 4a^2b^2c^2$
14. For what value of k is the function defined by  $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \pi/2 \\ 3, & \text{if } x = \pi/2 \end{cases}$  continuous at  $x = \pi/2$
15. For a positive constant a find  $dy/dx$ , where  $y = a^{t+(1/t)}$ , and  $x = [t + (1/t)]^a$

16. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 centimeters?

17. Evaluate:  $\int x\sqrt{x+x^2} dx$

OR

$$\int \frac{\tan^{-1}x}{(1+x)^2} dx$$

18. Evaluate:  $\int_0^\pi \frac{x}{(a^2 \cos^2 x + b^2 \sin^2 x)} dx$

19. Evaluate the following integral as limit of sum  $\int_0^4 (x + 3x^2 + e^{2x}) dx$

20. If the points (1,1,p) and (-3, 0, 1) be equidistant from the plane  $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$ , then find the value of p.

21. Show that  $(\vec{a} \times \vec{b})^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$

OR

If  $\vec{a}, \vec{b}, \vec{c}$  are unit vector such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$  then find the value of  $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$

22. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be hearts. Find the probability of missing card to be a heart.

**Section-C**

**[6×7 =42]**

23. Find the area of the region included between the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$ , where  $a > 0$ .

24. Solve the initial value problem  $e^{(dy/dx)} = x + 1; y(0)=5$

OR

Solve  $(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{2}{(x^2-1)}$

25. Show that the line whose vector equation is  $\vec{r} = (2\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 4\hat{k})$  is parallel to the plane

whose vector equation is  $\vec{r} \cdot (\hat{i} + 5\hat{j} + \hat{k}) = 5$ . Also, find the distance between them.

26. A random variable X has the following probability distribution:

$X_i :$	-2	-1	0	1	2	3
$P_i :$	0.1	k	0.2	2k	0.3	k

- (i) Find the value of k
- (ii) Calculate the mean of X
- (iii) Calculate the variance of X

27. A merchant plans to sell two types of personal computers - a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000.
28. An open box with a square base is to be made out of a given quantity of card board of the area  $c^2$  square units. Show that the maximum volume of the box is  $\frac{c^3}{6\sqrt{3}}$  cubic units.

OR

Separate the interval  $\left[0, \frac{\pi}{2}\right]$  into subintervals in which  $f(x) = (\sin^4 x + \cos^4 x)$  is

(a) increasing (b) decreasing

29. If  $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ , find  $A^{-1}$  and hence solve the system of linear equations

$$X + 2y + z = 4, \quad -x + y + z = 0, \quad x - 3y + z = 2$$

# CLASS XII

## GUESS PAPER-02

### MATHS

Time: 3Hours

Max.Marks: 100

#### Section-A

[1×10 =10]

1. Given a relation  $R = \{(2, 3), (1, 5), (3, 9)\}$ . Write  $R^{-1}$
2. Find the value of  $\sin^{-1}\left(\frac{x}{x+1}\right) + \sec^{-1}\left(\frac{x+1}{x}\right)$
3. Find the value of  $\lambda$ , such that Show that the vector  $2\hat{i} - \hat{j} + 4\hat{k}$  and  $\lambda\hat{i} + 3\hat{j} - \hat{k}$  are perpendicular.
4. Solve for  $x : \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = 0$
5. The total revenue  $R(x)$  received from the sale of  $x$  units is  $R(x) = 600x - \frac{x^3}{25}$ . Find the marginal revenue for  $x$  units.
6. Show that the distance between parallel planes  $2x - 2y + z + 3 = 0$  and  $4x - 4y + 2z + 5 = 0$  is  $\frac{1}{6}$
7. Equation of the plane is  $3x + 2y - 6z = 12$ . Find the intercepts cut the plane on axes.
8. Evaluate:  $\int \sin^2 x \, dx$
9. Find the area parallelogram whose sides are  $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ .
10.  $A$  is the non-singular matrix of order 3 and  $|A| = 9$ . Find  $|\text{adj } a|$

#### Section-B

[4×12 =48]

11. If  $\cos^{-1} a + \cos^{-1} b + \cos^{-1} c = \pi$ , then find that  $a^2 + b^2 + c^2 + 2abc = 1$ .
12. Show that the operation  $*$  on  $Z$ , defined by  $a * b = a + b + 1$  for all  $a, b \in Z$  satisfies
  - (i) the closer property,
  - (ii) the associative law and
  - (iii) the commutative law
  - (iv) Find the identity element in  $Z$
  - (v) What is inverse of an element  $a \in Z$ ?

13. Using properties of determinants, prove that  $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$
14. Test the continuity of the function  $f(x)$  at the origin =  $\begin{cases} \frac{x}{|x|}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$
15. If  $y = \sin^{-1}(x^2 \sqrt{1-x^2} + x \sqrt{1-x^4})$  then prove that  $\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}} + \frac{1}{\sqrt{1-x^2}}$
16. Using differentials, evaluate  $(82)^{1/4}$
17. Evaluate:  $\int \frac{\sin^{-1} x}{x^2} dx$
18. Evaluate:  $\int_0^{\pi/2} \log(\tan x) dx$
19. Evaluate  $\int_{-\pi/4}^{\pi/4} |\sin x| dx$
20. Find the image of the point (1, 6, 3) in the line  $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$
21. Find  $|\vec{a}|$  and  $|\vec{b}|$  if  $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 12$  and  $|\vec{a}| = 2|\vec{b}|$ .
22. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

**Section-C**

[6×7 =42]

23. Find the area of the region bounded by the curve  $y = \sqrt{1-x^2}$ , line  $y = x$  and the positive  $x$  - axis.
24. Solve the differential equation  $(y^2 - x^2)dy = 3xy dx$
25. Show that the equation of a plane, which meets the axes in A, B and C and the given centroid of triangle ABC is the point  $(\alpha, \beta, \gamma)$ , is  $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$
26. If the sum of the mean and variance of a binomial distribution for 5 trials is 1.8, find the distribution.
27. Solve the following linear programming problem graphically:
- Minimize  $Z = x - 5y + 20$ ,
- subjected to the constraints  $x - y \geq 0, -x + 2y \geq 2, x \geq 3, y \leq 4, x, y \geq 0$
28. Show that the rectangle of maximum area that can be inscribed in a circle of radius  $r$  is a square of a side  $\sqrt{2} r$ .
29. By using elementary transformations, find the inverse of the matrix  $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ .

# CLASS XII GUESS PAPER-03 MATHS

Time: 3Hours

Max. Marks: 100

### Section - A

[1×10 =10]

1. If A and B are symmetric matrices, show that AB is symmetric, if AB = BA.
2. If  $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$ , find a matrix B such that AB = I.
3. If  $A = \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}$ ,  $B = [3 \ 1 \ -2]$ , verify that  $(AB)' = B'A'$ .
4. Find the derivative of  $\sin^{-1}\left(\frac{2x}{x^2+1}\right)$  w.r.t. x
5. Show that the curves  $xy = a^2$  and  $x^2 + y^2 = a^2$  touch each other.
6. Evaluate  $\int_{-\pi/2}^{\pi/2} \sin^5 x \, dx$
7. Find the order and degree of the differential equation  $(dy/dx)^2 = 7x + (d^2y/dx^2)$
8. If  $|\vec{a}| = 5$ ,  $|\vec{b}| = 13$ ,  $|\vec{a} \times \vec{b}| = 25$ , Find  $\vec{a} \cdot \vec{b}$
9. If  $P(A) = 0.3$ ,  $P(B) = 0.5$  and  $P(A/B) = 0.4$ , find  $P(B/A)$ .
10. Two cards are drawn one by one without replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of aces.

### Section - B

[4×12 = 48]

11. Solve : (i)  $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$
12. Prove the following by the principle of mathematical Induction:  
 $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$  then  $A^n = \begin{bmatrix} 1 + 2n & -4n \\ n & 1 - 2n \end{bmatrix}$ , where n is any positive integer.
13. If  $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$ , prove that  $\frac{dy}{dx} = \frac{4}{1+x^2}$
14. Show that the area of the triangle formed by the tangent and normal at the point (a, a) on the curve  $y^2 = 2a - x$  =  $x^3$  and the line  $x = 2a$  is  $\frac{5a^2}{4}$  sq. units.
15. If  $f: [-5, 5] \rightarrow \mathbb{R}$  is differentiable function and if  $f'(x)$  does not vanish anywhere, then prove that  $f(-5) \neq f(5)$ .
16. Evaluate:  $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$

17. Evaluate:  $\int_0^{\sin^2 x} \sin^{-1} \sqrt{t} dt = \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} dt = \frac{\pi}{4}$
18. Prove by vector that the perpendiculars from the vertices to the opposite sides (i.e., altitudes) of a triangle are concurrent.
19. Show that  $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx = \frac{\pi^2}{4}$
20. Solve:  $x(1+y^2) dx - y(1+x^2) dy = 0$ , given that  $y = 0$  when  $x = 1$ .
21. A and B toss a coin alternately till one of them tosses a head and wins the game. If A starts the game, find their respective probabilities of winning.
22. An insurance company insured 2000 scooter drivers, 4000 car drivers' and 6000 truck drivers. The probability of an accident involving a scooter, car and truck is  $\frac{1}{100}$ ,  $\frac{3}{100}$ ,  $\frac{3}{20}$  respectively. One of the insured persons meets with an accident. What is the probability that he is a (i) scooter driver (ii) car drivers (iii) truck driver?

**Section - C**

**[6 × 7 = 42]**

23. Let  $A = Q \times Q$ . Let  $*$  be a binary operation on  $A$  defined by  $(a, b) * (c, d) = (ac, ad + b)$  Find (i) the identity element of  $(A, *)$  (ii) the invertible element of  $(A, *)$
24. Show that the height of the circular cylinder of maximum volume that can be inscribed in a given right- circular cone of height  $h$  is  $\frac{1}{3}h$ .
25. Find the area of the region bounded by the curves  $y = x + 2$ ,  $y = x^2$ ,  $x = 0$  and  $x = 3$ .
26. A farmer decides to plant up to 10 hectares with cabbages and potatoes. He decides to grow at least 2, but not more than 8 hectares of cabbages and at least 1 but not more than 6 hectares of potatoes. If he can make a profit of Rs. 1,500 per hectare on cabbages and Rs. 2,000 per hectare on potatoes, how should he plan his farming so as get the maximum profit? (Assuming that all the yield that he gets is sold).
27. Solve the following system of linear equations by matrix method,

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10 ;$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10;$$

$$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13 .$$

Where  $x \neq 0$ ,  $y \neq 0$ ,  $z \neq 0$

28. Find the distance of the point(1, -2, 3) from the plane  $x - y + z = 5$ , measured parallel to

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$$

29. Show that the lines  $\frac{x-1}{2} = \frac{y-3}{4} = -z$  and  $\frac{x-4}{3} = \frac{1-y}{2} = z-1$  are coplanar. Also find the equation of the plane containing the lines.

## CLASS XII GUESS PAPER-04 MATHS

Time: 3Hours

Max.Marks: 100

### Section - A

[1×10 =10]

1. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function defined as  $f(x) = 10x + 7$ . Find the function  $g: \mathbb{R} \rightarrow \mathbb{R}$  such that  $g \circ f = f \circ g = I_{\mathbb{R}}$ .
2. Write the range of one branch of  $\tan^{-1}x$ , other than the principal branch.
3. If A and B are symmetric matrices show that  $AB + BA$  is symmetric and  $AB - BA$  skew symmetric
4. Find the value of x for which  $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$
5. If  $\omega$  is a complex cube root of unity. Show that  $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$
6. Evaluate  $\int \frac{1}{x + \sqrt{x}} dx$
7. Evaluate  $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x} + \sqrt{10-x}} dx$
8. Find  $\vec{a} \cdot \vec{b}$  if  $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$  and  $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$ , then show that the vector  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  are perpendicular.
9. Show that  $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$
10. Show that the points A(1,2,7), B(2,6,3), C(3,10,-1) are collinear.

### Section - B

[4×12 = 48]

11. Show that the relation R in the set Z of integers given by  $R = \{(a, b) : 2 \text{ divides } a - b\}$  is an equivalence relation.
12. If  $\tan^{-1} \left\{ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right\} = \alpha$ , then prove that  $x^2 = \sin 2\alpha$



13. Prove that  $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

14. Find the value of k, such that the function  $f(x) = \begin{cases} k(x^2 - 2x), & \text{if } x < 0 \\ \cos x, & \text{if } x \geq 0 \end{cases}$  is continuous at  $x = 0$

15. If  $y = \sin\left(\frac{3x-4x^3}{x+1}\right) + x^{\tan x}$ , find  $\frac{dy}{dx}$

16. Using Lagrange's Mean Value Theorem, find a point on the parabola  $y = (x - 3)^2$ , where tangent is parallel to the chord joining (3, 0), (5, 4).

17. Evaluate  $\int \frac{(3\sin x - 2)\cos x}{5 - \cos^2 x - 4\sin x} dx$

18. Prove that  $|\vec{a} \cdot \vec{b}| \leq |\vec{a}||\vec{b}|$

19. Find the shortest distance between the lines  $l_1$  and  $l_2$  whose vector equations are  $\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$  and  $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \lambda(3\hat{i} - 5\hat{j} + 2\hat{k})$

20. A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probability of winning, if A starts first.

21. Find the general solution of the differential equation  $x \frac{dy}{dx} + 2y = x^2 (x \neq 0)$

22. Form the differential equation of the family of the circles touching the x-axis at origin.

### Section - C

[6 × 7 = 42]

23. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

24. Find the vector equation of the plane through the intersection of the plane  $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$  and  $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ , and the point (1, 1, 1).

25. Find the area of the smaller region bounded by the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and the line  $\frac{x}{3} + \frac{y}{2} = 1$

26. Evaluate:  $\int_0^2 (x^2 + 1) dx$ , as limit of sum.

27. Show that height of the cylinder of greater volume which can be inscribed in a right circular cone of height h and semi vertical angle  $\alpha$  is one-third that of the cone and the greatest volume is  $\frac{4}{27}\pi h^3 \tan^2 \alpha$ .

28. Obtain the inverse of the following matrix using elementary operations  $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

29. An aeroplane of an airline can carry a maximum of 200 passengers. A profit of Rs 400 is made on each first - class ticket and a profit of Rs 300 is made on each economy - class ticket. The airline reserves at least 20 seats for first class .However, at least 4 times as many passengers prefer to travel by economy class than by first class. Determine how many of each type of tickets must be sold in order to maximize the profit for the airline .What is the maximum profit?

## CLASS XII GUESS PAPER-05 MATHS

Time: 3Hours

Max.Marks: 100

Section - A

[1×10 =10]

1. Form a  $3 \times 4$  matrix  $A = [a_{ij}]$  where  $[a_{ij}]$  is given by  $[a_{ij}] = \frac{1}{2}|-3i + j|$ .
2. If A and B are symmetric matrices of the same order , then show that AB is symmetric if and only if A and B are commute, that is  $AB = BA$ .
3. If A is square matrix such that  $A^2 = A$ , then find the value of  $(I+A)^3 - 7A$
4. Differentiate,  $\sin x$  with respect to  $\log x$ .

5. Show that the function  $f$  given by  $f(x) = x^3 - 3x^2 + 4x$ ,  $x \in \mathbb{R}$  is strictly increasing on  $\mathbb{R}$ .

6. Evaluate  $\int \log x \, dx$

7. Write the order and degree of the differential equation  $y + x \frac{dy}{dy} = \sqrt{1 + \left(\frac{dy}{dy}\right)^2}$

8. Find a vector along the vector  $\hat{i} + 2\hat{j} + 3\hat{k}$  and has magnitude  $\sqrt{14}$  units.

9. If  $P(A) = 1/2$ ,  $P(B) = 1/3$  and  $P(A \cup B) = 2/3$ , Are the events  $A$  and  $B$  independent?

10. If  $A$  and  $B$  are mutually exclusive events, find  $P(A/B)$

**Section - B**

[4×12 = 48]

11. Show that  $\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{63}{16} = \pi$

12. Let  $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ , then prove that  $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

OR

Using properties of determinants, prove that  $\begin{vmatrix} a & a^2 & b+c \\ b & b^2 & c+a \\ c & c^2 & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$

13. If  $\cos y = x \cos(a+y)$ , with  $\cos a \neq \pm 1$ , prove that  $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ .

14. Prove that the curves  $x = y^2$  and  $xy = k$  cut at right angles if  $8k^2 = 1$

15. If  $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

find whether  $f(x)$  is continuous at  $x=0$

16. Evaluate:  $\int_0^{\pi} \frac{xdx}{a^2 \cos^2 x + b^2 \sin^2 x}$

17. Evaluate:  $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$

OR

Evaluate  $\int \frac{dx}{x(x^4 - 1)}$

18. Show that  $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$

19. Evaluate  $\int \frac{\sin x}{\sin(x-a)} dx$

OR

Evaluate  $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$

20. Solve the differential equation:  $\cos x \frac{dy}{dy} + y = \sin x$ , given that  $y = 2$  when  $x = 0$

OR

Solve the differential equation :  $(1+y^2)(1+\log x) dx + x dy = 0$  given that  $y = 1$  when  $x = 1$

21. Given three identical boxes I, II, III, each containing two coins. In box I, coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

22. Find the variance of the number obtained on a throw of an unbiased die.

**Section - C**

**[6 × 7 = 42]**

23. Given a non - empty set X, let  $*$  :  $P(X) \times P(X) \rightarrow P(X)$  be defined as  $A * B = (A - B) \cup (B - A) \forall A, B \in P(X)$ . Show that the empty set  $\phi$  is the identity for the operation  $*$  and all the elements A of  $P(X)$  are invertible with  $A^{-1} = A$ .

OR

Let  $f: N \rightarrow R$  be a function defined as  $f(x) = 4x^2 + 12x + 15$ . Show that  $f: N \rightarrow S$ , where s is the range of  $f$ , is invertible.

Find the inverse of  $f$ .

24. Show that the semi - vertical angle of the cone of the maximum volume and of given slant height is  $\tan^{-1} \sqrt{2}$ .

25. Find the area of the region enclosed between the two circles;

$x^2 + y^2 = 4$  and  $(x - 2)^2 + y^2 = 4$ .

26. Determine graphically the minimum value of the objective function

$$Z = -50x + 20y$$

Subject to the constraints

$$2x - y \geq -5;$$

$$3x + y \geq 3;$$

$$2x - 3y \leq 12;$$

$$x \geq 0, y \geq 0$$

27. Using matrix method, solve the following system of linear equations:

$$x + y + z = 4; 2x - y + z = -1; \quad 2x + y - 3z = -9$$

28. Find the equation of the plane which contains the line of intersection of the planes  $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$  and  $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ , and which is perpendicular to the plane  $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$

OR

Find the distance between the line  $l_1$  and  $l_2$  given by

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \text{ and } \vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

29. Show that the line  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and the line  $\frac{x-4}{5} = \frac{y-1}{2} = z$  intersect. Find the point of intersection also.

VK JHA M.Sc; B.Ed.

VK Education, 307, Sona – Cross Road Building, New C.G. Road, Chand -Kheda, Ahmedabad -382424

M: +919824168021

Email :vkedu@yahoo.com