CLASS XII
GUESS PAPER
MATHEMATICS

Time: 3 hrs M. M.: 100

Instructions: (i) All questions are compulsory.

(ii) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of 1 mark each, Section B comprises of 13 questions of 4 marks each, Section C comprises of 7 questions of 6 marks each.

(iii) There is no overall choice. However, internal choices have been provided in 4 questions of 4 marks each and 2 questions of six mark each.

SECTION 'A'

1. Find a unit vector in the direction of $\overline{AB}$, where A (1, 2, 3) and B (4, 5, 6) are the given points.
2. Let $\bar{a} = 2i + 3j + 2k$ and $\bar{b} = i + 2j + k$, find the projection of $\bar{a}$ on $\bar{b}$.
3. Find the Cartesian equation of the line which passes through the points (3, − 2, −5) and (3, −2, 6).
4. Construct a 2 X 3 matrix whose elements are given by $a_{ij} = \frac{1}{2}|5i - 3j|$
5. Form a differential equation for $x^2 + (y - b)^2 = 1$, where b is an arbitrary constant.

6. If m and n are the order and degree, respectively of the differential equation $y.(y_1)^3 + x^3.(y_2)^2 - x.y = \sin x$, then write the value of m + n.

SECTION 'B'

7. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags, scrap-books and pastel sheets made by them using recycled paper, at the rate of Rs. 20, Rs.15 and Rs. 5 per unit respectively. School A sold 25 paper-bags 12 scrap-books and 34 pastel sheets. School B sold 22 paper-bags, 15 scrapbooks and 28 pastel-sheets while school C sold 26 paper-bags, 18 scrap-books and 36 pastel sheets. Using matrices, find the total amount raised by each school. By such exhibition, which values are inculcated in the students?
8. If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for all n ∈ N.

OR

Using elementary row transformation, find the inverse of A = $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$.

9. If a, b, c is positive and unequal. Show that the value of determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative.

10. Evaluate: $\int_2^5 (|x-1| + |x-4|)\, dx$

11. Evaluate: $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$

OR

Evaluate: $\int (x+1)\sqrt{1-x-x^2}\, dx$

12. In a factory which manufactures bolts , machines A ,B and C manufacture respectively 25% ,35% and 40% of the bolts. Of their outputs 5% , 4% and 2% are respectively defective bolts .A bolt is drawn at random from the output and is found to be defective .What is the probability that it is manufacture by machine B .

OR

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

13. Let $\bar{a} = i - j$, $\bar{b} = 3j - k$ and $\bar{c} = 7i - k$. Find a vector $\bar{d}$ such that it is perpendicular to both $\bar{a}$ and $\bar{b}$ and $\bar{c}.\bar{d} = 1$.

14. Find the image of the point (2, 3, - 4) in the plane $\bar{r}.(2i - j + k) = 3$.

OR

Find the distance between the point P (6, 5, 9) and the plane determined by the points A (3, -1, 2), B (5, 2, 4) and C (-1, -1, 6).

15. Prove that: $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{cb+1}{b-c}\right) + \cot^{-1}\left(\frac{ac+1}{c-a}\right) = 2\pi$, if $a < b < c$

OR

Solve $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{2x-1}{2x+1} = \tan^{-1}\frac{23}{36}$.

16. If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, find d^2y/dx^2.

17. If $y = x^x - 2^{\sin x}$, then find dy/dx.

18. State and verify Lagrange's mean value (LMV) theorem $f(x) = x^3 - 2x^2 - x + 3$ on [0, 1].

19. Evaluate: $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\, dx$.

SECTION 'C'

20. Let f: W → W is defined as f (n) = n – 1, if n is odd and f (n) = n + 1, if n is even. Show that f is invertible. Find the inverse of f.

OR

Consider the function $f : R_+ \to [4, \infty)$ defined by $f(x) = x^2 + 4$, where R_+ is the set of all non – negative real numbers, show that f is invertible. Also, find the inverse of f.

21. Using integration, find the area of enclosed figure by

{ (x, y) : $0 \leq y \geq x^2 + 1$; $0 \leq y \geq x + 1$; $0 \leq x \geq 2$ } .

22. Solve the differential equation: $2\, y.e^{x/y}\, dx + (y - 2\, x.e^{x/y})\, dy = 0$.

OR

Solve the differential equation: $(x - \sin y)\, y' + \tan y = 0$; $y(0) = 0$

23. Find the shortest distance between the lines whose vector equations are $\bar{r} = (1 - t)i + (t - 2)j + (3 - 2t)k$ and $\bar{r} = (s + 1)i + (2s - 1)j - (2s + 1)k$. And also find its equation.

24. 40% students of a college reside in hostel and the remaining resides outside. At the end of year, 50% of the hosteliers got A grade while from outside students, only30% got A grade in the examination. At the end of year, a student of the college was chosen at random and was found to get A grade. What is the probability that the selected student was a hostelier?

25. An open box with a square base is to be made out of a given card board of area c^2 sq. units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

26. David wants to invest at most Rs. 12000 in bonds A and B. According to the rule, he has to invest at least Rs. 2000 in bond A and at least Rs. 4000 in bond B. If the rates of interest on bond A and B are 8% and 10% per annum respectively. Formulate the LPP and solve it graphically for maximum interest. Also, determine the maximum interest received in a year. Why investment is important for future life?