CLASS XII
GUESS PAPER
MATHEMATICS

Time Allowed : 3 Hours **Maximum Marks : 100**

General Instructions : *The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.*

SECTION – A

1. If $f : R \to R$ be defined by $f(x) = (3 - x^3)^{1/3}$, then find $fof(x)$.

2. Find the area of the triangle whose vertices are P(1, 1), Q(2, 7) and R(10, 8).

3. Find the principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.

4. Evaluate : $\int \sin 3x \cos 4x \, dx$.

5. Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

6. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, show that $A^{-1} = \frac{1}{19}A$.

7. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$

8. Find the value of λ if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

9. Evaluate : $\int_0^{\pi} \frac{x}{1 + \sin x} dx$

10. Find the direction cosines of the line passing through the points (–2, 4, –5) & (1, 2, 3).

SECTION – B

11. Let * be a binary operation defined on $N \times N$, by $(a,b) * (c,d) = (a + c, b + d)$. Show that * is commutative and associative. Also find the identity element for * on $N \times N$.

OR

Show that the relation R in the set $A = \{x : x \in W, 0 \le x \le 12\}$ given by $R = \{(a,b) : (a,\ b)$ is a multiple of 4} is an equivalence relation. Also, find the set of all elements related to 2.

12. Prove that : $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = (bc+ca+ab+abc)$

13. Solve of x : $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}, \quad |x|<1$

OR

Solve for x : $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)+\tan^{-1}\left(\frac{2x}{x^2-1}\right)=\frac{2\pi}{3}$.

14. For what value of k is the function $f(x)=\begin{cases} \frac{1-\cos 4x}{8x^2}, & if\ x\neq 0 \\ k, & if\ x=0 \end{cases}$ is continuous at $x = 0$?

15. Differentiate $x^{\sin x}+(\sin x)^{\cos^{-1}x}$ with respect to x.

OR

If $y=\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$, find $\frac{dy}{dx}$.

16. Solve : $(x^2+xy)dy=(x^2+y^2)dx$

17. Two balls are drawn from an urn containing 2 white, 3 red and 4 black balls one by one without replacement. What is the probability that at least one ball is red?

18. Evaluate : $\int \frac{1}{\sqrt{8+2x-x^2}}\,dx$

19. Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{d}_1=3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{d}_2=\hat{i}-3\hat{j}+4\hat{k}$.

20. Solve : $\frac{dy}{dx}+\frac{y}{x}=e^x$

21. If $y=e^{a\cos^{-1}x}, -1\le x\le 1$, show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$.

OR

If $y=(\tan^{-1}x)^2$, show that $(x^2+1)^2y_2+2x(x^2+1)y_1=2$.

22. Find the shortest distance between the lines, whose equations are $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{10-z}{-7}$ and $\frac{x-15}{3}=\frac{58-2y}{-16}=\frac{z-5}{-5}$.

SECTION – C

23. Using matrices solve the system of equations : $x+2y-3z=-4;\ 2x+3y-3z=2;\ 3x-3y-4z=11$

24. Find the equation of the plane passing through the point A(1, 1, −1) perpendicular to the planes $x+2y+3z-7=0$ and $2x-3y+4z=0$.

25. Using integration, find the area of the region : $\{(x,y); 9x^2+y^2\le 36 \text{ and } 3x+y\ge 6\}$

26. Evaluate : $\int_1^4 [|x-1|+|x-2|+|x-4|]dx$

OR

Evaluate : $\int \frac{x^4}{(x-1)(x^2+1)}dx$

27. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

OR

Show that the height of a closed cylinder of given volume and the least surface area is equal to its diameter.

28. A manufacturer produces two types of steel trunks. He has two machines A and B. the first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type of trunk requires 3 hours on machine A and 2 hours on machine B. Machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs 30 and Rs 25 per trunk on the first type and second type respectively. How many trunks of each type must be make each day to make the maximum profit.

29. A letter is known to have come either from TATANAGAR or CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that letter has been come from (i) CALCUTTA and (ii) TATANAGAR.

OR

Find the probability distribution of the number of white balls drawn in a random draw of 3 balls without replacement from a bag containing 4 white and 6 red balls. Also find the mean and variance of the distribution.

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