Roll No.
रोल नं.

Series DPS

Code No. /1
कोड नं. /1

- **Please check that this question paper contains 0 printed pages.**
- **Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.**
- **Please check that this question paper contains 29 questions.**
- **Please write down the serial number of the question before attempting it.**
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 0 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 29 प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें।

MATHEMATICS
गणित

Time allowed : 3 hours
निर्धारित समय : 3 घण्टे

Maximum Marks : 100
अधिकतम अंक : 100

***General Instructions**: -*

i) All the questions are compulsory.

*ii) The question paper consists of 29 questions divided into **three** sections **A, B** and **C**. Section **A** contains **10** questions of **1** mark each, Section **B** contains **12** questions of **4** marks each and Section **C** contains 7 questions of **6** marks each.*

*iii) There is no overall choice. However an internal choice in any 4 questions of **four** marks each and any two questions of six marks each has been provided.*

iv) Use of Calculator is not permitted. You may ask for logarithmic tables, if required.

Section – A

Questions number 1 to 10 carry 1 marks each:

1. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then find A^2 hence find A^6.
2. For what value of a, $\begin{pmatrix} 2a & -1 \\ -8 & 3 \end{pmatrix}$ is a singular matrix.
3. Write the number of all one-one functions from the set A = {a, b, c} to itself.
4. Find the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z+3}{-6}$ is perpendicular to the plane $3x-y-2z = 7$.
5. Evaluate $\int_0^1 \left[\{x\}\right] dx$ .Where {.} is fractional part and [.] is greatest integer function.
6. Given $\overrightarrow{AB} = 3\hat{i} - \hat{j} - \hat{k}$ and coordinates of the terminal point are (0, 1, 3). Find the coordinates of the initial point.
7. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on vector $7\hat{i} - \hat{j} + 8\hat{k}$.
8. If $f'(x) = \frac{1}{x} + x$ and $f(1) = \frac{5}{2}$, then what is $f(x)$?
9. Find the value of $arc\ \sin\left(\sin\frac{2\pi}{3}\right)$.
10. Without expanding the determinant show that: $\begin{vmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{vmatrix} = 0$.

Section-B

Questions number 11 to 22 carry 4 marks each.

11. From a pack of 52 playing cards, a card is accidently dropped. From the remaining 51 cards two cards are drawn at random (without replacement) and are found to be both spades. Find the probability that the dropped card was a card of club?
12. Find whether the lines $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} + \hat{j})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect or not. If they intersect, find the point of intersection. If do not intersect, find the shortest distance between them.

OR

Prove that if a plane has the intersepts a, b, c and is at a distance of p units from the origin, then $a^{-2}+b^{-2}+c^{-2}=p^{-2}$

13. $\vec{a}, \vec{b}, \vec{c}$ are the unit vectors. Suppose $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a}=\pm 2(\vec{b}\times\vec{c})$.

OR

If $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-5\hat{k}$, then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are orthogonal.

14. Evaluate: $\int \frac{1}{x^4+1}dx$.

15. Let $A=N\times N$. Let $*$ be binary operation on A defined by $(a,b)*(c,d)=(ad+bc,bd)$. Then
(i) find the identity element of $(A,*)$
(ii) is $(A,*)$ commutative?

16. Solve for x, $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$.

OR

Prove that: $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$.

17. Prove by using properties of determinants:

$$\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3.$$

18. For what value of a and b, the function defined as:

$$f(x)=\begin{cases} 3ax+b & ;\ if \quad x<1 \\ 11 & ;\ if \quad x=1 \\ 5ax-2b & ;\ if \quad x>1 \end{cases}$$ is continuous at $x=1$.

19. If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that $\frac{\left[1+(y')^2\right]^{3/2}}{y''}$ is a constant and free from a and b.

OR

If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\dfrac{dy}{dx}=\sqrt{\dfrac{1-y^2}{1-x^2}}$.

20. Find the interval in which the function *f* given by $f(x)=\sin^4 x+\cos^4 x$ is strictly increasing and strictly decreasing.

21. Solve the differential equation: $\dfrac{dy}{dx}-\dfrac{1}{x}.y=2x^2$.

22. Solve: $(x^3+x^2+x+1)\dfrac{dy}{dx}=2x^2+x$.

Section-C

Question number 23 to 29 carry 6 marks each:

23. Using integration find the area of the region included between the parabola $y^2=x$ and the line $x+y=2$.

OR

Using definite integration, find the area of the region: $\left\{(x,y):|x-1|\le y\le\sqrt{5-x^2}\right\}$.

24. A bag contains 4 balls. Two balls are drawn at random, and are found to be blue. What is the probability that all the balls are blue?

25. Find the equation of the line of intersection of planes $4x+4y-5z=12$ and $8x+12y-13z=32$ in the vector and symmetric form.

26. A window is in the shape of a rectangle surmounted by a semicircle. If its perimeter is 30 m, then find the dimensions of the window so that it may admit maximum light.

27. Evaluate $\int_1^4 (x^2-x)dx$ as the limit of a sum.

28. If $A=\begin{bmatrix}2 & 1 & 3\\ 4 & -1 & 0\\ -7 & 2 & 1\end{bmatrix}$ find A^{-1}, and hence solve the following system of equations:

$$\begin{aligned} 2x + y + 3z &= 3 \\ 4x - y \quad &= 3 \\ -7x + 2y + z &= 2. \end{aligned}$$

OR

Obtain the inverse of the following matrix using elementary transformations: $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$.

29. There are two factories, one located at Vidyut Nagar and the other in Delhi, from these locations, a certain number of machines are to be delivered to each of the three depots situated at P, Q and R. The weekly requirements of the depots are respectively 5, 5, and 4 units of the machines while the production capacity of the factories at Vidyut Nagar and Delhi are 8 and 6 units respectively. The cost of transportation per unit is given below.

From ↓ / To →	Cost (in Rs.)		
	P	Q	R
Vidyut Nagar	160	100	150
Delhi	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum?
What is the minimum transportation cost?