

D.V.R'S INSTITUTE OF MATHEMATICS
CHENNAI – 600 050 (9940116934)
XII CBSE - MATHEMATICS MODEL PAPER
2004 – 2005 - SET - 2

SECTION – A (1 x 6 = 6 Marks)

- Solve $\begin{vmatrix} 3 & x \\ 2 & 4 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 2 & 4 \end{vmatrix}$
- Find the integrating factor of $dx + xdy = e^{-y}\sec^2ydy$.
- Find the order and degree of the differential equation $5y\left(\frac{dy}{dx}\right)^2 + \sin x\left(\frac{dx}{dy}\right) = 3x$
- Find the area of the parallelogram determined by one diagonal $\hat{i} + 2\hat{j} + 3\hat{k}$ and one side $3\hat{i} - 2\hat{j} + \hat{k}$.
- Find the position vector of the point which divides the join of the points $2\vec{a} - 3\vec{b}$ and $3\vec{a} - 2\vec{b}$ externally in the ratio 2:3
- Find the intercepts made by the plane $\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 5$ on the co-ordinate axes.

SECTION – B (13 x 4 = 42 Marks)

- Show that the relation R in the set R of real numbers, defined as $R = \{(a,b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

(OR)

Define a binary operation * on the set $A = \{0,1,2,3,4,5\}$ as

$$a * b = \begin{cases} a + b & \text{if } a + b < 6 \\ a + b - 6 & \text{if } a + b \geq 6 \end{cases} \quad \text{(i) Prepare a composition table (ii) Show that } (A, *) \text{ is}$$

associative, (iii) Find the identity element, (iv) Find the inverse of every element.

- Find the value of $\tan^{-1} \left[\sin^{-1} \left[\frac{2x}{1+x^2} \right] + \cos^{-1} \left[\frac{1-y^2}{1+y^2} \right] \right]$, $|x| < 1$, $y > 0$ and $xy < 1$

- Find the values of a and b if $f(x) = \begin{cases} \frac{1 - \sin^3 x}{3 \cos^2 x}; & x < \frac{\pi}{2} \\ a & ; x = \frac{\pi}{2} \\ \frac{b(1 - \sin x)}{(\pi - 2x)^2}; & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{4}$

- If $y\sqrt{x^2+1} = \log[\sqrt{x^2+1}-x]$, Show that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$

(or)

If $x = a(\cos\theta + \theta\sin\theta)$; $y = a(\sin\theta - \theta\cos\theta)$, find d^2y/dx^2

- Verify Rolle's theorem for $f(x) = \sin^4x + \cos^4x$ in $[0, \pi/2]$

12. Find intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is (a) strictly increasing (b) strictly decreasing

13. Evaluate $\int \frac{dx}{\sqrt{3x^2 + 6x + 12}}$

14. Evaluate $\int_{-1}^{\frac{3}{2}} |x \sin \pi x| dx$

15. Show that four points with position vectors $6\hat{i} - 7\hat{j}$, $16\hat{i} - 19\hat{j} - 4\hat{k}$, $3\hat{j} - 6\hat{k}$ and $2\hat{i} + 5\hat{j} + 10\hat{k}$ are coplanar.

16. Solve $(3xy + y^2)dx + (x^2 + xy)dy = 0$.
(OR)

Solve the following differential equation : $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

17. Find the particular solution of the differential equation $(1 - x^2)\frac{dy}{dx} - xy = x^2$, given that $y(0) = 2$.

18. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

(OR)

Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 7$.

19. A pair of dice is thrown 7 times. If getting a total of 7 is considered a success, what is the probability of (i) no success (ii) at least 6 success (iii) at most 6 success.

SECTION - C (7 x 6 = 42 Marks)

20. In a Legislative assembly election, a political party hired a public relation firm to promote its candidate in three ways; telephone, house calls and letters. The numbers of contacts of each type in three cities A, B & C are (500, 1000, and 5000), (3000, 1000, 10000) and (2000, 1500, 4000), respectively. The party paid Rs. 3700, Rs.7200, and Rs.4300 in cities A, B & C respectively. Find the costs per contact using matrix method. Keeping in mind the economic condition of the country, which way of promotion is better in your view?

21. Prove the following by principle of mathematical induction, If $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$ then,

$$A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix} \text{ for every positive integer } n$$

22. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius r is $\frac{2r}{\sqrt{3}}$.

(OR)

A given quantity of metal is to be cast into a solid half circular cylinder (i.e., with rectangular base and semicircular ends). Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its circular ends is $\pi : (\pi + 2)$

23. Make a rough sketch of the region given below and find the area using the method of integration : $\{(x, y); 0 \leq y \leq x^2 + 3, 0 \leq y \leq 2x + 3, 0 \leq x \leq 3\}$
24. Find the distance of the point $A(-2, 3, -4)$ from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.
25. An NGO is helping the poor people of earthquake hit village by providing medicines. In order to do this they set up a plant to prepare two medicines A and B. There is sufficient raw material available to make 20000 bottles of medicine A and 40000 bottles of medicine B but there are 45000 bottles into which either of the medicine can be put. Further it takes 3 hours to prepare enough material to fill 1000 bottles of medicine A and takes 1 hour to prepare enough material to fill 1000 bottles of medicine B and there are 66 hours available for the operation. If the bottle of medicine A is used for 8 patients and bottle of medicine B is used for 7 patients. How the NGO should plan his production to cover maximum patients? How can you help others in case of natural disaster?
26. In a school, 30% of the student has 100% attendance. Previous year result report tells that 70% of all students having 100% attendance attain A grade and 10% of remaining students attain A grade in their annual examination. At the end of the year, One student is chosen at random and he has an A grade. What is the probability that the student has 100% attendance? Also state the factors which affect the result of a student in the examination.

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