Minimum preparation for passing (for slow learners)

RELATIONS AND FUNCTIONS	5 Marks
INVERSE TRIGONOMETRIC FUNCTIONS	5 Marks
MATRICES AND DETERMINANTS	13 Marks
VECTOR ALGEBRA	7 Marks
3-D GEOMETRY	10 Marks
LINEAR PROGRAMING	6 Marks
BAYE'S THEOREM	6 Marks
TOTAL	**52 Marks**

CLASS XII

QUESTION BANK

RELATIONS AND FUNCTIONS (5 Marks)

1- Marks (1 question in board)

1. If $f(x) = x^2 +1$ and $g(x) = 3x + 1$, then find fog and gof
2. Let $f:\{2,3,4,5\}\to\{3,4,5,9\}$ and $g:\{3,4,5,9\}\to\{7,11,15\}$ be functions defined as $f(2) =3$, $f(3)=4$, $f(4)=5$, $f(5)=5$ and $g(3)=g(4)=7$ and $g(5) = g(9) = 11$. find gof.
3. Let $*$ be a binary operation defined on Q. such that $a*b=ab/4$ find $5*4$,
4. Let $*$ be a binary operation defined on Q. such that $a*b = a–b+ab$ find $3*4$,
5. Let $*$ be a binary operation defined on Q. such that $a*b = ab^2$, find $4*3$
6. State how many number of binary operations are possible on the set { a, b }.

7. Check whether the following $*$ are binary operations, If not state reason:
   (i) On Z^+, define $*$ by $a*b = a-b$ (ii) On Z^+, define $*$ by $a*b = |a-b|$
   (i) On R, define $*$ by $a*b = ab^2$

8. If f : R→R be given by f(x) = $\left(3-x^3\right)^{\frac{1}{3}}$ then find fof(x)

9. State with reason whether the following functions have inverse
   (i) $f:\{1,2,3,4\}\rightarrow\{10\}$ with $f=\{(1,10),(2,10),(3,10),(4,10)\}$
   (ii) $g:\{5,6,7,8\}\rightarrow\{1,2,3,4\}$ with $g=\{(5,4),(6,3),(7,4),(8,2)\}$
   (iii) $h:\{2,3,4,5\}\rightarrow\{7,9,11,13\}$ with $h=\{(2,7),(3,9),(4,11),(5,13)\}$

10. Check whether f is one to one and onto (i) $f : R \to R$ be defined as $f(x) = x^4$,
    (ii) $f : R \to R$ be defined as $f(x) = 3x$.

4- Marks(1 question in board)

1. Let $*$ be a binary operation on the set Q of rational numbers as follows:(i) $a * b = a - b$, (ii) $a * b = a^2 + b^2$, (iii) $a * b = a + ab$, (iv) $a * b = (a - b)^2$, (v) $a * b = a\ b^2$.
   Which of these binary operation are commutative and which are associative.
2. Check whether the binary operation $*$ defined on Z such that $a*b=a+b-15$ is
   i) commutative (ii) associative.(iii) Find the identity element. (iv) Find the inverse of an element.
3. Define a binary operation $*$ on the set A = {0,1,2,3,4,5} as
   $a*b = \begin{cases} a+b \text{ if } a+b<6 \\ a+b-6 \text{ if } a+b\geq 6 \end{cases}$ Show that zero is the identity for this operation and each element 'a' of the set is invertible with 6-a being the inverse of 'a'..
4. Consider f:N→N, g:N→Nand h:N→R where f(x) = 2x, g(y) = 3y+4, h(z) = sinz, ∀ x,y,z∈N.. Show that ho(gof) = (hog)of.
5. Show that the relation R defined by (a,b) R (c,d) ⇒ a+d = b+c on the set NxN is an equivalence relation.
6. Let f:N →N be defined by f(n) $\begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ \frac{n}{2}, \text{ if } n \text{ is even} \end{cases}$, for all n∈N. State whether the function f is bijective, justify your answer.

7. Show that if $f : R-\left\{\frac{7}{5}\right\} \to R-\left\{\frac{3}{5}\right\}$ is defined by $f(x) = \frac{3x+4}{5x-7}$ and $g : R-\left\{\frac{3}{5}\right\} \to R-\left\{\frac{7}{5}\right\}$ is defined by $f(x) = \frac{7x+4}{5x-3}$ then fog = I_A and gof = I_B, where. $A = R-\left\{\frac{3}{5}\right\}$, $B = R-\left\{\frac{7}{5}\right\}$, I_A (x) = x on A , I_B (x) = x on B are called identity functions on sets A and B respectively.
8. T is a set of triangles and relation R:T→T is given by R = {(Δ_1, Δ_2) ∈TxT / $\Delta_1 \equiv \Delta_2$ }, Show that R is an equivalence relation.
9. Show that the relation R in the set R of real numbers, defined as R = {(a,b) : $a \le b^2$ } is neither reflexive nor symmetric nor transitive.
10. Let R be the relation on N XN defined by (a,b)R(c,d) ⇔ad=bc for all (a,b), (c,d) ∈NXN. Show that R is an equivalence relation on NXN.
11. Let A = NxN and ∗ be the binary operation on A defined by (a,b) ∗ (c, d) = (a+c, b+d) . Show that (i) (A, ∗) is associative, (ii) (A,∗) is commutative (iii) Find the identity element if exists.
12. Show that the function f:N-N given by f(x) = x^2 is injective and not surjective.

INVERSE TRIGONOMETRIC FUNCTIONS (5 – MARKS)

1- Marks(1 question in board)

1. Find the principal value of (i) $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$, (ii) $\cos^{-1}\left(-\frac{1}{2}\right)$, (ii) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$
2. Find the value of $\cos\left(\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right)$
3. Find the value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$
4. Write the range of $\sin^{-1}x$.
5. Show that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x$

6. Write the following in the simplest form

(i) $\tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), |x|>1$ (ii) $\cos^{-1}\left(\frac{1-x}{x+1}\right)$, (iii) $\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right), |x|<a$

4- Marks(1 question in board)

1. Simplify $\tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right); x \neq 0$
2. Simplify $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$
3. Prove that (i) $\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$,
4. Prove that (ii) $\sin^{-1}\left(\frac{12}{13}\right)+\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\frac{63}{16}=\pi$
5. Find the value of $\tan^{-1}\frac{1}{2}\left(\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right), |x|<1, y>0, xy<1$
6. Prove that $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$,
7. Prove that $\cot^{-1}\left[\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right]=\frac{x}{2}$ where $0 \leq x \leq \frac{\pi}{4}$
8. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$, where $\frac{-1}{\sqrt{2}} \leq x \leq 1$
9. Solve the following equations for x: $\tan^{-1}\frac{x+1}{x-1}+\tan^{-1}\frac{x-1}{x}=\tan^{-1}(-7)$
10. Solve for x (iii) $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$,
11. Solve for x if $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

MATRICES AND DETERMINANTS (13 Marks)

1-Marks (3 questions in board)

1. Construct a 3x2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i-3j|$

2. Construct a 3x2 matrix whose elements are given by $a_{ij} = \frac{(i+2j)^2}{2}$

3. Find x and y if $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix} + \begin{bmatrix}y & 0\\1 & 2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

4. Find the matrix X such that $X\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix} = \begin{pmatrix}-7 & -8 & -9\\2 & 4 & 6\end{pmatrix}$

5. If A = $\begin{pmatrix}1 & a\\0 & 1\end{pmatrix}$ then A^{50} is - - - -

6. The number of all possible matrices of order 3 x 3 with each entry 0 or 1 is - - - -

7. If A,B,C are three non zero square matrices of same order, find the condition on A such that AB = AC $\Rightarrow$ B = C.

8. If B is a skew symmetric matrix, write whether the matrix (ABA') is symmetric or skew symmetric.

9. If A is of order 3 x 4 and BA is of order 2 x4, then the order of B is - - - -

10. Solve $X\begin{pmatrix}5 & 3\\-1 & -2\end{pmatrix} = \begin{pmatrix}14 & 7\\7 & 7\end{pmatrix}$

11. If for a matrix A, $|adjA| = 64$ where A is a 3^{rd} order square matrix, then find $|3A|$

12. If $x\begin{bmatrix}2\\3\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$, find the value of x and y..

<u>4-Marks (1 question in board)</u>

1. Let A = $\begin{pmatrix}3 & 7\\2 & 5\end{pmatrix}$ and B = $\begin{pmatrix}6 & 8\\7 & 9\end{pmatrix}$, Verify that $(AB)^{-1}=B^{-1}A^{-1}$.

2. Let A = $\begin{pmatrix}3 & 1\\-1 & 2\end{pmatrix}$ show that $A^2 - 5A + 7I = 0$. Hence findA^{-1}.

3. Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ 4 \\ 1 \end{pmatrix} = 0$

4. If A = $\begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$.

5. If A = $\begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$, then verify that A (*adj* A)=$|A|$I Also find A^{-1}

6. If $A = \begin{pmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{pmatrix}$, then find the matrix X, such that 2A + 3X = 5B.

7. Find X and Y if X + Y = $\begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix}$ and X-Y = $\begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$.

8. If A = $\begin{pmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{pmatrix}$, B = $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$, C = $\begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$ Calculate AC, BC and (A+B)C. Also verify that (A+B)C = AC+ BC

9. If A = $\begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$, then show that $A^3 - 23A - 40I = 0$

10. Express the matrix B = $\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

11. Find the inverse of $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$

12. Show that $\begin{vmatrix} a+b+c & -c & -b \\ -c & b+c+a & -a \\ -b & -a & c+a+b \end{vmatrix} = 2(a+b)(b+c)(c+a)$

13. If $x \neq y \neq z$, and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that xyz = –1.

14. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

15. Prove that $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$ = (a–b)(b–c)(c–a)(a+b+c)

6-Marks (1 question in board)

1. Show that $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

2. Prove without expanding $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + ab + bc + ca = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

3. Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$

4. $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$. Verify that $A^3 - 6A^2 + 9A - 4I = O$. Hence find A^{-1}.

5. Solve the following system of equations by using matrix inversion method. $\frac{2}{x}+\frac{3}{y}+\frac{10}{z} = 4; \frac{4}{x}-\frac{6}{y}+\frac{5}{z} = 1; \frac{6}{x}+\frac{9}{y}-\frac{20}{z} = 2$

6. The sum of three numbers is 6. If we multiply third number by 3 and second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

7. If $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$ *and* $B=\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ find AB. Use this to solve the following system of equations. x–y+2z=1, 2y–3z = 1 and 3x–2y+4z = 2.

8. If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations 2x–3y+5z = 11; 3x+2y–4z = –5; x+y–2z = –3.

9. Let A = $\begin{pmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{pmatrix}$ and I is the identity matrix of order 2, show that $I+A=(I-A)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}$.

10. Prove the following by principle of mathematical induction, If $A=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$ then, $A^n=\begin{pmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \text{con } n\theta\end{pmatrix}$ for every positive integer n.

11. Prove the following by principle of mathematical induction, If $A=\begin{pmatrix}3 & -4\\ 1 & -1\end{pmatrix}$ then, $A^n=\begin{pmatrix}1+2n & -4n\\ n & 1-2n\end{pmatrix}$ for every positive integer n.

12. Let $A=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, show that $(aI+bA)^n=a^nI+na^{n-1}bA$ where I is the identity matrix of order 2 and n∈N

13. If A = $\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$, , prove that $A^n=\begin{pmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\end{pmatrix}$, n ∈ N

VECTOR ALGEBRA (7 Marks)

1-Mark (3 question in board vectors and 3-D)

1. $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them,

   then $(\vec{a}+\vec{b})$is a unit vector if - - - -

2. If $\vec{a}$ and $\vec{b}$ include an angle of 120 degrees and their magnitude are 2 and $\sqrt{3}$ then $\vec{a}\cdot\vec{b}$ is - - - -

3. If $\vec{a}+\vec{b}+\vec{c}=0, |\vec{a}|=3, |\vec{b}|=4, |\vec{c}|=5,$ then the angle between $\vec{a}$ and $\vec{b}$ is - - - -

4. If a line makes 45, 60, with positive direction of x-axis x, y then the angle made with the z axis is - - -

5. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\vec{i}-\vec{j}+3\vec{k}$ *and* $\vec{b}=2\vec{i}-7\vec{j}+\vec{k}$.

6. The area of the parallelogram having a diagonal $3\vec{i}+\vec{j}-\vec{k}$ and a side $\vec{i}-3\vec{j}+4\vec{k}$ -

7. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then - - -

8. Find the angle between the vectors $3\vec{i}-2\vec{j}+6\vec{k}$ and $4\vec{i}-\vec{j}+8\vec{k}$.
9. Find the projection of $7\vec{i}+\vec{j}-4\vec{k}$ *on* $2\vec{i}+6\vec{j}+3\vec{k}$.
10. If $\vec{p},\vec{q}$ *and* $\vec{p}+\vec{q}$ are vectors of magnitude λ, then $|\vec{p}-\vec{q}|$ is - -

11. Find the direction cosines of the line passing through the points (1, 3,–2) and (2, –2, 3).

12. Two lines one passing through (2, 4,–1) and (2, k, 2) and another passing through (1, 3,–2) and (2, –2, 3). Find k so that the line are perpendicular.

13. Find the vector equation and Cartesian equation of the line through the point (3,–4,–2) and parallel to the vector $9\vec{i}+6\vec{j}+2\vec{k}$.

14. Find the vector and Cartesian equation of the straight line passing through the points (–5,2,3) and (4,–3,6).

15. Find the angle between the planes $\vec{r}\cdot(2\vec{i}-\vec{j}-\vec{k})=15$ and $\vec{r}\cdot(-\vec{i}+\vec{j}-3\vec{k})=3$.

16. Find the angle between the line $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{-2}$ and the plane 3x+4y+z+5=0.

17. Find the equation of the line parallel to $\frac{x-3}{1}=\frac{y+3}{5}=\frac{2z-5}{3}$ and passing through the point (1,3,5) in vector form.

4-Marks (1 question in board)

1. A girl walks 4km towards west, then she walks 3km in a direction 30^0 east of north and stops. Determine the girl's displacement from her initial point of departure.
2. If $\vec{a}x\vec{b}=\vec{c}x\vec{d};\ and\ \vec{a}x\vec{c}=\vec{b}x\vec{d}$, prove that $(\vec{a}-\vec{d})$ is parallel to $(\vec{b}-\vec{c})$ provided $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.
3. Show that the points A(1,–2,–8),B(5,0,–2) and C(11,3,7) are collinear, and find the ratio in which B divides AC.
4. Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k},\ \vec{b}=\hat{i}+2\hat{j}+3\hat{k}$.
5. If $\vec{a}=\hat{i}+4\hat{j}+2\hat{k},\ \vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$ .Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}\cdot\vec{d}=15$.
6. Show that the vector $\vec{i}+\vec{j}+\vec{k}$ is equally inclined to x, y, and z axes.
7. Show that the vectors $\vec{a}=3\vec{i}-4\vec{j}-4\vec{k},\vec{b}=2i-\vec{j}+\vec{k}\quad and\ \vec{c}=\vec{i}-3\vec{j}-5\vec{k}$, respectively form the vertices of a right angled triangle.
8. If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=8$, then find $|\vec{x}|$.
9. If $\vec{a},\vec{b}\ and\ \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$
10. Show that the points A(1, 2, 7) , B(2,6,3) , C(3,10,-1) are collinear.
11. Find the position vector of a point R which divides the line joining the points P and Q whose position vectors are $2\vec{a}+\vec{b}\ and\ \vec{a}-3\vec{b}$ externally in the ratio 1:2. Also show that P is the midpoint of the line segment RQ.
12. With usual notation prove that $a^2= b^2+c^2–2bc\cos A$.
13. If $\vec{a},\vec{b}$ and $\vec{c}$ are the position vectors of the vertices of the triangle ABC, find the expression for the area of the triangle ABC and deduce the condition for the points A,B,C to be collinear.
14. Find the area of the triangle whose vertices are A(1,1,1), B(1,2,3) and C(2,3,1).

15. If $\hat{a}$ and $\hat{b}$ are unit vectors and θ is the angle between them, then prove that
(i) $\sin\frac{\theta}{2} = \frac{1}{2}|\hat{a}-\hat{b}|$ (ii) $\cos\frac{\theta}{2} = \frac{1}{2}|\hat{a}+\hat{b}|$
16. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of equal magnitude, prove that the angle which $(\vec{a}+\vec{b}+\vec{c})$ makes with any of the vectors $\vec{a}, \vec{b}$ or $\vec{c}$ is $\cos^{-1}\frac{1}{\sqrt{3}}$
17. If $\vec{d}_1$ and $\vec{d}_2$ are the diagonals of a parallelogram with sides $\vec{a}_1$ and $\vec{a}_2$. Find the area of the parallelogram in terms of $\vec{d}_1$ and $\vec{d}_2$, and hence find the area with $\vec{d}_1 = \hat{i}+2\hat{j}+3\hat{k}$ and $\vec{d}_2 = 3\hat{i}-2\hat{j}+\hat{k}$
18. If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, $\vec{\alpha} = 3\hat{i}-\hat{j}$, $\vec{\beta} = 2\hat{i}+\hat{j}-3\hat{k}$, then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.
19. Prove by vector methods in triangle ABC $a = b\cos C + c\cos B$

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3-D GEOMETRY (10 MARKS)

4-Marks (1 question in board)

1. Find the shortest distance between the parallel lines $\vec{r} = (\vec{i}-\vec{j}) + s(2\vec{i}-\vec{j}+\vec{k})$ and $\vec{r} = (2\vec{i}+\vec{j}+\vec{k}) + s(2\vec{i}-\vec{j}+\vec{k})$.
2. Find the shortest distance between the skew lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$
3. Find the equation of the plane passing through the point (2,1,3) and the line of intersection of the planes $\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k}) = 7$ and $\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k}) = 9$.
4. Find the equation of the plane passing through the intersection of the planes x+y+z =1 and 2x+3y+4z=5= 0 and perpendicular to the plane x–y+z=0
5. Find the vector and Cartesian equation of the plane passing through the points (1,1,–1), (6,4,–5) and (–4,–2,3).

6. Derive the equation of the plane in the intercept form.
7. Find the vector and Cartesian equation of the plane passing through the points A(0,0,0) and B(3,–1,2) and is parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$.
8. Find the vector and Cartesian equation of the plane passing through (–1. 3 . 2) point and perpendicular to the planes x+2y+3z=5 and 3x + 3y +z = 0
9. Find the meeting point of the line $\vec{r}=(2\vec{i}+\vec{j}-3\vec{k})+t(2\vec{i}-\vec{j}-\vec{k})$ and the plane x–2y+3z+7=0.
10. Find the meeting point of the line $\vec{r}=(2\vec{i}+\vec{j}-3\vec{k})+t(2\vec{i}-\vec{j}-\vec{k})$ and the plane x–2y+3z+7=0.
11. Find the foot of perpendicular from the point (1,6,3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.

6-Marks (1 question in board)

1. Find the vector equation to the plane through the point (–1,3,2) and perpendicular to the planes x+2y+2z=5 and 3x+y+2z=8.
2. Find the length of the foot of perpendicular drawn from the point (2,–1,5) to the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$. Also find its image.
3. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{1}$ intersect. If so find the point of intersection of the lines
4. Find the distance of the point (–2,3,–4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane 4x+12y–3z+1 = 0.
5. Find the co-ordinates of the image of the point (1,3,4) in the plane 2x – y + z + 3 = 0.
6. Find the distance of the point (–1,–5,–10) from the point of intersection of the line $\vec{r}=(2\vec{i}-\vec{j}+2\vec{k})+t(3\vec{i}+4\vec{j}+2\vec{k})$ and the plane $\vec{r}\cdot(\hat{i}-\hat{j}+k)=5$.
7. Find the image of a point (1,6,3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. Also find the image.
8. Find the distance of the point (2,3,4) from the plane 3x+2y+2z+5=0 measured parallel to the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$

9. A variable plane is at a constant distance 3p from the origin and meets the axes in A,B,C respectively, then show that locus of the centroid of triangle ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$

LINEAR PROGRAMING (6-Marks)

1. A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

2. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs17.50 per package on nuts and Rs 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day?

3. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs 4 per unit food and F_2 costs Rs 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

4. There are two types of fertilisers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

5. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of

cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

6. A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for atleast 5 hours a day. She produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs 600 and Rs 400 on items M and N respectively. How many of each item should she produce so as to maximise her profit assuming that she can sell all the items that she produced? What will be the maximum profit?

7. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

8. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?

9. If a young man rides his motorcycle at 25km/hr, he had to spend Rs.2 per km on petrol. If he rides at a faster speed of 40km/hr, the petrol cost increases at 5 per km. He has Rs.100 to spend on petrol and wishes to find what is the maximum distance, he can travel in one hour. Express this as an LPP and solve it graphically.

10. Anil wants to invest at most Rs.12000 in Bonds A and B. According to the rules, he has to invest at least Rs.2000 in Bond A and at least Rs.4000 in bond B. If the rate of interest on Bond A is 8% per annum and on bond B is 10% per annum, how should he invest his money for maximum interest? Solve the problem graphically.

BAYE'S THEOREM (6 MARKS)

1. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time.
2. Bag I contains 3 red and 4 black balls while another Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag II.
3. Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?
4. Suppose that the reliability of a HIV test is specified as follows: Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV–ive but 1% are diagnosed as showing HIV+ive. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her as HIV+ive. What is the probability that the person actually has HIV?
5. In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?
6. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively 3/10 ,1/5, 1/10 and 2/5. The probabilities that he will be late are 1/4, 1/3 ,and 1/12 , if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?
7. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.
8. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of

the insured persons meets with an accident. What is the probability that he is a scooter driver?

9. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.
10. If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly setup.
11. Coloured balls are distributed in four boxes as shown in the following table:

Box	Colour			
	Black	White	Red	Blue
I	3	4	5	6
II	2	2	2	2
III	1	2	3	1
IV	4	3	1	5

A box is selected at random and then a ball is randomly drawn from the selected box. The colour of the ball is black, what is the probability that ball drawn is from the box III?

WISH YOU ALL THE BEST

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