

CLASS XII GUESS PAPER MATHEMATICS

Time 3.00 Hrs.

M.M. 100

Instructions:

- (i) The paper consists of 3 section A,B,C .
- (ii) Section A contain 10 questions of 1 marks each and section B contain 12 questions of 4 marks each and section C contain 7 questions of 6 marks each.
- (iii) All questions are compulsory. DO all questions in serial order.

Section A

- Q1. Find a unit vector parallel to the sum of the vectors $a = 2i + 4j - 5k$ and $b = i + 2j + 3k$
- Q2. Prove that : $\tan^{-1}(-1) + \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{\pi}{2}$.
- Q3. If $\vec{a} = 2i + 3j - k$, $\vec{b} = i + j + 2k$, find angle between them.
- Q4. If $A = \begin{bmatrix} 1 & 3 \\ -1 & 4 \end{bmatrix}$, find $|adjA|$
- Q5. If $\lambda \neq 0$ and $\begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix} = 0$, then find the value of x.
- Q6. Evaluate the approximate value of $\sqrt{0.37}$.
- Q7. Evaluate $\int \log x \, dx$.

- Q8. If f and g be given by $f = \{(1,-1), (2,-4), (3, -9)\}$ & $g = \{(-1,3), (-4,7), (-9,11)\}$ Show that $g \circ f$ is defined but $f \circ g$ is not defined .
- Q9. Find the eq of the plane which is normal to the vector $i + j + k$ and passes through the point $(2,3,4)$
- Q10. If B is a skew symmetric matrix, write whether the matrix (ABA') is symmetric or Skew symmetric matrix

Section B

- Q11. If a, b and c are real numbers and $\begin{vmatrix} b+c & c+a & a+b \\ a+c & a+b & b+c \\ a+b & c+b & a+c \end{vmatrix} = 0$ then show that $a + b + c = 0$ or $a = b = c$

Q12. Evaluate : $\int \frac{x^2+1}{(x+1)^2} dx$

Q13. If $y = x^x$, show that $\frac{d^2y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx} \right)^2 - \frac{y}{x} = 0$

OR

If $y\sqrt{x^2+1} = \log(\sqrt{x^2+1} - x)$, then prove that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$

- Q14. Find the probability distribution of the number of heads in a single throw of three coins and find the mean of this distribution.
- Q15. Show that the relation R in the set $A = \{x : x \in W, 0 \leq x \leq 12\}$ given by $R = \{(a,b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Also find the set of all elements related to 2
- Q16. Find the shortest distance between the lines $r = i + j + \lambda(2i - j + k)$ and $r = 2i + j - k + \mu(3i - 5j + 2k)$

Q17. Express the vector $5\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ as the sum of two vectors such that one is parallel and other is perpendicular to the vector $3\mathbf{i} + \mathbf{k}$.

Q18. If $\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = \pi$, then prove that $\alpha\beta\gamma = \alpha + \beta + \gamma$
OR

$$\text{Solve : } \cos^{-1} \frac{x^2 - 1}{x^2 + 1} + \tan^{-1} \frac{2x}{x^2 - 1} = \frac{2\pi}{3}$$

Q19. Prove that : $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, if $f(-x) = f(x)$
 $= 0$ if $f(-x) = -f(x)$

Q20. Evaluate: $\int \frac{1}{\cos x + \operatorname{cosec} x} dx$

Q21. Water is leaking from the funnel at the rate of $5\text{cm}^3/\text{sec}$. If the radius of the base of funnel is 5 cm. and height 10 cm. find the rate at which the water level is dropping when it is 2.5 cm. from the top.

OR

Verify whether Lagrange's mean value theorem is applicable for $f(x) = x(x - 1)(x - 2)$ in $[0, 1/2]$, and find c

Q22. Find a, b, c if $f(x)$ is continuous at $x = 0$, $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}, & x > 0 \end{cases}$

Section C

Q23. If $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$, Find A^{-1} , use it to solve the following system of equations: $2x - 3y + 5z = 16$, $3x + 2y - 4z = -4$, $x + y - 2z = -3$

- Q24. Find the eq of the plane passing through the line of intersection of the planes $x - 2y + z = 1$ and $2x + y + z = 8$ and parallel to the line with direction ratio $1, 2, 1$. Also find the distance of $P(1, -2, -2)$ from this plane measured along a line parallel to $r = t(i - 2j - 5k)$
OR
Prove that the image of the point $(3, -2, 1)$ in the plane $3x - y + 4z = 2$ lies on the plane $x + y + z + 4 = 0$.
- Q25. A dietician has to develop a special diet using two foods P and Q. Each packet of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and atmost 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet? What is the min amount of vitamin A ?
- Q26 Sketch the graph $f(x) = \begin{cases} |x-2|+2, & x \leq 2 \\ x^2-2, & x > 2 \end{cases}$ Evaluate $\int_0^4 f(x)dx$ What does the value of this integral represent on the graph?
OR
Make a rough sketch of the region given below and find it's area using method of integration: $\{(x,y): 0 \leq y \leq x^2+3, 0 \leq y \leq 2x+3, 0 \leq x \leq 3\}$
- Q27 Solve the equation :
a) $(3xy + y^2)dx + (x^2 + xy)dy = 0$ b) $e^x \sqrt{1-y^2} dx + \frac{y}{x} dy = 0$
- Q28 A square tank of capacity 250 cubic metres has to be dug out. The cost of the land is Rs 50 per sq meter. The cost of digging increases with the depth and for the whole tank it is Rs $400h^2$, where h metres is the depth of the tank. What should be the dimension of the tank so that the cost be minimum ?
- Q29 A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be hearts. Find the probability of the missing card to be a heart.