

CLASS XII GUESS PAPER MATHEMATICS

Time : 3 Hours

Maximum Marks : 100

GENERAL INSTRUCTIONS : The question paper consists of 29 questions divided into three sections A, B & C. Section A comprises of 10 questions of 1 mark each, section B comprises of 12 questions of 4 marks each & section C comprises of 7 questions of 6 marks each.

SECTION – A

1. Evaluate : $\int \frac{x + \cos 6x}{3x^2 + \sin 6x} dx$.
2. A matrix A is of order 2×2 has determinant 4. What is the value of $|2A|$?
3. Write the value of $\int_0^{\pi} \cos^5 x dx$.
4. Write the value of $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{k} \times \hat{i}) + \hat{k} \cdot (\hat{j} \times \hat{i})$.
5. If $\vec{a} = \hat{i} + 2\hat{j}$, $\vec{b} = \hat{j} + 2\hat{k}$, write a unit vector along the vector $3\vec{a} - 2\vec{b}$.
6. Write the angle between the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{-2}$ and the plane $x + y + 4 = 0$.
7. If A is a square matrix, write a condition on A such that $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$.
8. If A and B are non-singular matrices of the same order, then write the relationship between $\text{adj } AB$, $\text{adj } A$ and $\text{adj } B$.
9. Find the principal value of $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$.
10. Let $*$ be a binary operation defined by $a * b = a - b + ab$. Find $7 * 3$.

SECTION – B

11. Solve the differential equation: $(x + y)dy + (x - y)dx = 0$, given that $y(1) = 1$.
12. Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f : A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$. Is f one-one and onto? Justify your answer.

- 13.** Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$
- 14.** If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$, $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$.
- 15.** Solve the differential equation : $\frac{dy}{dx} + \sec x \cdot y = \tan x \left(0 \leq x \leq \frac{\pi}{2}\right)$.
- 16.** The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 centimeters?
- OR**
- Find the intervals in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing or decreasing. Also find the points at which the tangents are parallel to the x-axis.
- 17.** Find the image of point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.
- 18.** Using properties of determinants, prove that :
- $$\begin{vmatrix} 3a & -a+b & -a+c \\ a-b & 3b & c-b \\ a-c & b-c & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca).$$
- 19.** Find all points of discontinuity of f , where $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ x+1, & \text{if } x \geq 0 \end{cases}$.
- 20.** A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize
(a) at least once (b) exactly once (c) at least twice?
- OR**
- In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.
- 21.** Evaluate : $\int \frac{1}{\sqrt{4x-3x^2+5}} dx$.
- OR**
- Evaluate : $\int_0^{\pi/4} \sin 2x \sin 3x dx$.
- 22.** If $x \sin(a+y) + \sin a \cos(a+y) = 0$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.
- OR**

Differentiate the following with respect to x : $\sin^{-1} \left[\frac{\sqrt{1+x} - \sqrt{1-x}}{2} \right]$.

SECTION – C

- 23.** Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A(2, 0), B(4, 5) and C(6, 3).
- 24.** Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.
- OR**
- Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimetres, find the dimensions of the can which has the minimum surface area?
- 25.** Evaluate : $\int_0^{\pi/4} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$.
- 26.** Find the coordinates of the foot of the perpendicular drawn from the point A(1, 8, 4) to the line joining the points B(0, -1, 3) and C(2, -3, -1).
- 27.** Using matrices, solve the following system of equations : $x + 2y + z = 7$, $x + 3z = 11$ and $2x - 3y = 1$.

OR

Using elementary row-transformations, find the inverse of $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

- 28.** A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs. 30 while by selling one table the profit is Rs. 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P. and solve it graphically.
- 29.** In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?