Series : PTS/19

Code No. 16/12/19

Roll No. | 2 | 1 | 6 | 1 | 2 | | |

Candidates must write the Code on the title page of the answer-book.

PLEASURE TEST SERIES XII – 19

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Time Allowed : 180 Minutes **Max. Marks : 100**

SECTION A

Q01. Evaluate the integral : $\int \frac{\sec x \tan x + \sec^2 x}{(\sec x + \tan x)^n} dx$. **Q02.** Find $\int_{-2}^{2} \frac{x^2}{1+5^x} dx$.

Q03. For what value of x, the vectors $(x-2)\vec{a} + \vec{b}$ and $(2x+1)\vec{a} - \vec{b}$ are collinear? Assume that the vectors $\vec{a}$ and $\vec{b}$ are non-collinear.

Q04. If a function $f(x)$ is differentiable at $x = c$, then what is the value of $\lim_{x \to c} f(x)$?

SECTION B

Q05. (a) If $y = 2^{\sin(x - \cos x)}$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$. (b) Differentiate $\tan(x^\circ + 60^\circ)$ w.r.t. x.

Q06. Let A = $[a_{ij}]$ be a square matrix of order 3 and C_{ij} denotes the cofactor of $[a_{ij}]$ in A. If |A| = 5, then write the value of $a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$.

Q07. Evaluate $\int_{98}^{100} (x-98)(x-99)(x-100) dx$.

Q08. The income I of Dr. Priya Goel is given by I(x) = ₹ $(x^3 - 3x^2 + 5x)$. Can an insurance agent ensure her for the growth of her income? Justify.

Q09. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\vec{a} - \vec{b}$ to be a unit vector?

Q10. Let f : {1, 2, 3} $\to$ {a, b, c} given by $f(1) = a, f(2) = b$ and, $f(3) = c$. Show that $(f^{-1})^{-1} = f$.

Q11. Find the distance of a point (2, 5, –3) from the plane $\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$.

Q12. Let E_1 and E_2 be two independent events such that $P(E_1) = p_1$ and $P(E_2) = p_2$.
Describe in words of the events whose probabilities are :
(i) p_1p_2 (ii) $(1-p_1)p_2$ (iii) $1-(1-p_1)(1-p_2)$ (iv) $p_1 + p_2 - 2p_1p_2$.

SECTION C

Q13. If $y = \cos^{-1}\sqrt{\frac{\cos 3x}{\cos^3 x}}$ then prove that $\frac{dy}{dx} = \sqrt{\frac{3}{\cos 3x \cos x}}$.

Q14. Find the particular solution of differential equation cos x dy = sin x (cos x – 2y)dx, given that $y = 0$ when x = $\pi/3$.

Q15. If $x = \operatorname{cosec}[\tan^{-1}\{\cos(\cot^{-1}\sec(\sin^{-1} a))\}]$ and $y = \sec[\cot^{-1}\{\sin(\tan^{-1}\operatorname{cosec}(\cos^{-1} a))\}]$, then find a relation between x and y in terms of a.

Q16. Evaluate $\int \frac{f^3}{f^6 + 1} df$. **OR** Evaluate $\int \frac{1}{\sin 2x + \sqrt{3}\cos 2x} dx$.

Q17. Last year, 1 packet of tea and 3 packets of sugar together cost ₹96. This year, the rate of tea increased by 15% and that of sugar by 10%. So, the same amounts of tea and sugar now cost ₹108.60. Find the rates of sugar and tea per packet last year and this year using matrix method.

OR A university gives scholarships for those students who take any of the below subjects as an additional subject in first year, second year, third year of graduation. From the table given below, form a set of simultaneous equations and check the consistency.

Sr. No.	Subject	No. of students in 1^{st} year	No. of students in 2^{nd} year	No. of students in 3^{rd} year
1.	Industrial Waste	1	3	6
2.	Organic Waste	1	1	7
3.	e-Waste	1	1	8
	Amount Received	₹5000	₹7000	₹35800

Q18. The lengths of the sides of an isosceles triangle are $9 + x^2$, $9 + x^2$ and $18 - 2x^2$ units. Calculate the area of the triangle in terms of x and find the value of x which makes the area maximum.

OR Find the least value of 'a' such that the function $f(x) = x^2 + 2ax + 3$ is strictly increasing on (3, 4).

Q19. Discuss the continuity of $f(x) = \begin{cases} 2+\sqrt{1-x^2}, & x \le 1 \\ 2e^{(1-x)^2}, & x > 1 \end{cases}$ at x = 1.

Q20. A and B take turn in throwing two dice. The first to throw 9 is being awarded. Show that if A has the first throw, their chances of winning are in the ratio 9:8.

Q21. For any two vectors $\vec{a}$ and $\vec{b}$, show that : $(1+|\vec{a}|^2)(1+|\vec{b}|^2) = (1-\vec{a}.\vec{b})^2 + |\vec{a}+\vec{b}+\vec{a}\times\vec{b}|^2$.

Q22. Two bikers are running at the speed more than allowed speed on the road along the lines $\vec{r} = \hat{i}+\hat{j}-\hat{k}+\lambda(3\hat{i}-\hat{j})$ and $\vec{r} = 4\hat{i}-\hat{k}+\mu(2\hat{i}+3\hat{k})$. Using Shortest Distance formula, check if they meet to an accident or not?

Q23. An urn contains 4 white and 6 red balls. Four balls are drawn at random (without replacement) from the urn. Find the probability distribution of number of white balls.

SECTION D

Q24. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also, find the equation of the plane containing these two lines.

Q25. Determine graphically the minimum and maximum values of the objective function $Z = -50x + 20y$ subject to the constraints : $2x - y \ge -5$, $3x + y \ge 3$, $2x - 3y \le 12$, $x \ge 0$, $y \ge 0$.

Q26. Show that $\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)$.

OR Prove that $(x-2)(x-1)$ is factor of $\begin{vmatrix} 1 & 1 & x \\ \beta+1 & \beta+1 & \beta+x \\ 3 & x+1 & x+2 \end{vmatrix}$. Hence, write the quotient.

Q27. Consider a relation R in the set A of people in a colony defined as aRb iff a and b are members of joint family. Is R is an equivalence relation? Give reason (s).

OR Let A and B be two sets. Show that $f : A \times B \to B \times A$ s. t. $f(a,b) = (b,a)$ is a bijection.

Q28. Using integrals, find the area of ΔABC whose vertices have the coordinates as A(–1, 1), B(0, 5) and C(3, 2).

OR Evaluate area of the region enclosed between the curves $y^2 = x+1$ and $y^2 = -x+1$.

Q29. Find $\int x(\log x)^2 \, dx$.

SOLUTIONS & MARKING SCHEME for PTS – 19 [2016 - 2017]

SECTION A

Q01. $\dfrac{(\sec x+\tan x)^{1-n}}{1-n}+C$. **Q02.** 16/3.

Q03. 1/3. **Q04.** $f(c)$

SECTION B

Q05. (a) $\dfrac{dy}{dx}=2^{\sin(x-\cos x)}\log 2\times\cos(x-\cos x)(1+\sin x)$

$\therefore \left.\dfrac{dy}{dx}\right]_{\text{at } x=\frac{\pi}{2}}=2^{\sin\left(\frac{\pi}{2}-\cos\frac{\pi}{2}\right)}\log 2\times\cos\left(\dfrac{\pi}{2}-\cos\dfrac{\pi}{2}\right)\left(1+\sin\dfrac{\pi}{2}\right)=0$.

(b) Let $y=\tan\left(\dfrac{\pi x}{180}+\dfrac{\pi}{3}\right)$ $\therefore \dfrac{dy}{dx}=\dfrac{\pi}{180}\sec^2(x^\circ+60^\circ)$. **Q06.** 5.

Q07. Use property $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$ to get $\int_{98}^{100}(x-98)(x-99)(x-100)dx=0$.

Q08. Show that $\dfrac{d}{dx}[\,I(x)\,]>0$ for all x. **Q09.** 30°. **Q11.** 13/7 units.

Q12. (i) $p_1p_2=P(E_1)P(E_2)=P(E_1\cap E_2)$

$=P$ (simultaneous occurrence of E_1 and E_2)

(ii) $(1-p_1)p_2=P(\overline{E}_1)P(E_2)=P(\overline{E}_1\cap E_2)=P(E_2-E_1)=P(E_2$ but not $E_1)$

(iii) $1-(1-p_1)(1-p_2)=1-P(\overline{E}_1)P(\overline{E}_2)=1-P(\overline{E}_1\cap\overline{E}_2)$

$=1-\{1-P(E_1\cup E_2)\}=P(E_1\cup E_2)$

$=P$ (at least one of E_1 and E_2 occurs)

(iv) $p_1+p_2-2p_1p_2=(p_1-p_1p_2)+(p_2-p_1p_2)$

$=\{P(E_1)-P(E_1\cap E_2)\}+\{P(E_2)-P(E_1\cap E_2)\}$

$=P(E_1\cap\overline{E}_2)+P(E_2\cap\overline{E}_1)$

$=P$ (exactly one of E_1 and E_2 occurs).

SECTION C

Q13. Here $y=\cos^{-1}\sqrt{\dfrac{\cos 3x}{\cos^3 x}}$ $\Rightarrow \cos y=\sqrt{\dfrac{\cos 3x}{\cos^3 x}}$ $\Rightarrow \cos^2 y=\dfrac{\cos 3x}{\cos^3 x}=\dfrac{4\cos^3 x-3\cos x}{\cos^3 x}$

$\Rightarrow \cos^2 y=4-3\sec^2 x$ $\therefore -\sin 2y\dfrac{dy}{dx}=0-6\sec^2 x\tan x$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{6\sin x}{2\sin y\cos y\cos^3 x}$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{3\sin x}{\sin y\sqrt{\dfrac{\cos 3x}{\cos^3 x}}\cos^3 x}$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{3\sin x}{\sin y\sqrt{\cos^3 x\cos 3x}}$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{3\sin x}{\sqrt{1-\cos^2 y}\sqrt{\cos^3 x\cos 3x}}$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{3\sin x}{\sqrt{3\sec^2 x-3}\sqrt{\cos^3 x\cos 3x}}$ $\Rightarrow \dfrac{dy}{dx}=\dfrac{3\sin x}{\sqrt{3}\sqrt{\dfrac{1-\cos^2 x}{\cos^2 x}}\sqrt{\cos^3 x\cos 3x}}$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{3}}{\sqrt{\frac{\cos^3 x \cos 3x}{\cos^2 x}}} \qquad \therefore \frac{dy}{dx} = \sqrt{\frac{3}{\cos 3x \cos x}}.$$

Q14. We have cos x dy = sin x (cos x – 2y)dx $\Rightarrow \frac{dy}{dx} + (2\tan x)y = \sin x$

It is linear diff. eq. of the form $\frac{dy}{dx} + P(x)y = Q(x)$. $\therefore P(x) = 2\tan x, Q(x) = \sin x$

I.F. $= e^{\int 2\tan x\, dx} = \sec^2 x$. So solution is given by : $y(\sec^2 x) = \int \sec^2 x \sin x\, dx + C$

i.e., $y(\sec^2 x) = \sec x + C$. And $\because$ $y = 0$ when $x = \frac{\pi}{3}$ so, $0 \times \left(\sec^2 \frac{\pi}{3}\right) = \sec\frac{\pi}{3} + C \quad \Rightarrow C = -2$

Hence required solution is : $y = \cos x - 2\cos^2 x$.

Q15. $x = y = \sqrt{3 - a^2}$.

Q16. Let $I = \int \frac{f^3}{f^6 + 1} df = \int \frac{f^2 \times f}{(f^2)^3 + 1} df \quad \left[\text{Put } f^2 = x \Rightarrow f df = \frac{dx}{2}\right.$

$\therefore I = \frac{1}{2}\int \frac{x}{x^3 + 1} dx \qquad \Rightarrow I = \frac{1}{2}\int \frac{x}{(x+1)(x^2 - x + 1)} dx$. Now use Partial Fraction.

OR Let $I = \int \frac{1}{\sin 2x + \sqrt{3}\cos 2x} dx = \frac{1}{2}\int \frac{1}{\frac{1}{2}\sin 2x + \frac{\sqrt{3}}{2}\cos 2x} dx$

$$\Rightarrow I = \frac{1}{2}\int \frac{1}{\cos\left(2x - \frac{\pi}{6}\right)} dx = \frac{1}{2}\int \sec\left(2x - \frac{\pi}{6}\right) dx = \frac{1}{4}\log\left|\sec\left(2x - \frac{\pi}{6}\right) + \tan\left(2x - \frac{\pi}{6}\right)\right| + C.$$

Q18. Area : 6x (9 – x^2) Sq. units, $x = \sqrt{3}$ **OR** $a = -3$.

Q19. Continuous at x = 1.

Q20. Getting 9 means getting 'sum of the nos. on the dice as 9'.
Let E : sum of the nos. on the dice as 9 so, $E = \{(3,6),(4,5),(5,4),(6,3)\}$

That is, $P(E) = \frac{4}{36} = \frac{1}{9}$ and, $P(\bar{E}) = 1 - \frac{1}{9} = \frac{8}{9}$.

As A has the first throw so, he may win in 1st or 3rd or 5th or, ... throws.

Therefore, $P(\text{A wins}) = P(E) + P(\bar{E})P(\bar{E})P(E) + P(\bar{E})P(\bar{E})P(\bar{E})P(\bar{E})P(E) + ...$

$$\Rightarrow P(\text{A wins}) = \frac{1}{9} + \left(\frac{8}{9}\right)^2 \times \frac{1}{9} + \left(\frac{8}{9}\right)^4 \times \frac{1}{9} + ... = \frac{\frac{1}{9}}{1 - \frac{64}{81}} = \frac{9}{17} \text{ and, } P(\text{B wins}) = 1 - P(\text{A wins}) = \frac{8}{17}.$$

Hence $P(\text{A wins}) : P(\text{B wins}) = \frac{9}{17} : \frac{8}{17} = 9 : 8$.

Q21. See Vol.2 of Mathematicia *by* O.P. Gupta

Q22. S.D. = 0, this means they meet to an accident.

SECTION D

Q24. $x - 2y + z = 0$.

Q25. See NCERT Example 4

Q26. Let $\Delta = \begin{vmatrix} -a(b^2 + c^2 - a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2 + a^2 - b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2 + b^2 - c^2) \end{vmatrix}$

Taking a, b, c common from C_1, C_2 and C_3 respectively.

$$\Delta = abc\begin{vmatrix} -b^2-c^2+a^2 & 2b^2 & 2c^2 \\ 2a^2 & -c^2-a^2+b^2 & 2c^2 \\ 2a^2 & 2b^2 & -a^2-b^2+c^2 \end{vmatrix} \quad \text{By } C_1 \to C_1 + C_2 + C_3,$$

$$\Delta = abc\begin{vmatrix} a^2+b^2+c^2 & 2b^2 & 2c^2 \\ a^2+b^2+c^2 & -c^2-a^2+b^2 & 2c^2 \\ a^2+b^2+c^2 & 2b^2 & -a^2-b^2+c^2 \end{vmatrix} \quad \text{Taking } a^2+b^2+c^2 \text{ common from C1,}$$

$$\Delta = abc(a^2+b^2+c^2)\begin{vmatrix} 1 & 2b^2 & 2c^2 \\ 1 & -c^2-a^2+b^2 & 2c^2 \\ 1 & 2b^2 & -a^2-b^2+c^2 \end{vmatrix}. \text{ Now complete.}$$

OR Qotient $= \beta$.

Q27. Yes, R is an equivalence relation.

OR We've $f : A \times B \to B \times A$ such that $f(a,b) = (b,a)$

One-one : Let (a,b) & $(c,d) \in A \times B$ such that $a \neq c, b \neq d\ \forall a, c \in A$ and $b, d \in B$.

Then $f(a,b) = (b,a)$ & $f(c,d) = (d,c) \Rightarrow (b,a) \neq (d,c)$ $[\because b \neq d,\ a \neq c$

That means, $f(a,b) \neq f(c,d)$ $\therefore f$ is one-one…(i)

Onto : For all $a \in A,\ b \in B$, we have $(b,a) \in B \times A$ which also implies $(a,b) \in A \times B$

$\therefore f$ is onto …(ii)

By (i) and (ii), we can conclude that f is bijective function.

Q28. 15/2 sq. units. **OR** 8/3 Sq. units.

Q29. $\dfrac{x^2}{4}\left[2(\log x)^2 - 2\log x + 1\right] + C$.