FIRST PRELIMINARY EXAMINATION – JAN 2018

TIME: 3 Hrs

STD XII **MATHEMATICS** **MARKS: 100**

GENERAL INSTRUCTIONS:

1) All questions are compulsory.
2) This question paper contains 29 questions.
3) Question 1-4 in Section A are very short- answer type questions carrying 1 mark each.
4) Question 5-12 in section B are short -answer type questions carrying 2 marks each.
5) Question 13-23 in Section C are long -answer -I type questions carrying 4 marks each.
6) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

SECTION A

1. Let A = {1, 2, 3, 4}. Let R be the equivalence relation on A x A defined by (a, b)R(c, d) iff a + d = b + c. Find the equivalence class [(1,3)].
2. If A is a square matric and |A| = 2, then write the value of $\left|AA^t\right|$, where A^t is the transpose of A
3. If $\vec{a}\ and\ \vec{b}$ are perpendicular vectors such that $\left|\vec{a}+\vec{b}\right| = 13\ and \left|\vec{a}\right| = 5$, find the value of $\left|\vec{b}\right|$
4. If A ={3,5,7} and B = {2,4,9} and R is a relation from A to Bgiven by "is less than", then write R as a set of ordered pairs.

SECTION B

5. Find x, if $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$
6. If A = $\begin{pmatrix} 0 & 3 \\ -2 & 5 \end{pmatrix}$, then find k, so that $kA^2 = 5A - 6I$
7. Solve $\cos\left(\tan^{-1}x\right)=\sin\left(\cot^{-1}\frac{3}{4}\right)$
8. Prove that the tangents to the curve $y = 2x^3 - 4$ at the points x = 2 and x = -2 are parallel.

9. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

10. Verify $ax^2 + by^2 = 1$ is a solution of the differential equation $x(yy_2 + y_1^2) = yy_1$

11. Find the projection of $2\hat{i} - \hat{j} + \hat{k}$ on $\hat{i} - 2\hat{j} + \hat{k}$

12. If X has a binomial distribution $B\left(4, \frac{1}{3}\right)$, then find P(x = 1)

SECTION C

13. Using properties of determinants, show that $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

14. Find the value of a and b, if the function given by f(x) = $\begin{cases} ax^2 + b, & if \ x < 1 \\ 2x+1, & if \ x \geq 1 \end{cases}$ is differentiable at x = 1

OR

If f(x) = $\begin{cases} ax+b & if \quad x > 2 \\ 7x-4 & if \quad x = 2 \\ 3ax-2b & if \quad x < 2 \end{cases}$ is a continuous function at x = 2 find the value of a and b.

15. If y = $x^{\cot x} + (\sin x)^x$, find $\frac{dy}{dx}$

16. Find the intervals in which the function f(x) = $2x^3 - 8x^2 + 10x + 5$ is strictly increasing or strictly decreasing.

OR

The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3cm per second. How fast is the area decreasing when the two equal sides are equal to the base?

17. Show that the height of a closed right circular cylinder of given surface and maximum volume is equal to the diameter of the base.

18. Evaluate $\int \frac{2x^2 + 5x + 7}{(x-2)(x-3)^2} dx$

19. Show that (x - y)dy = (x + 2y)dx is a homogeneous differential equation. Also, find the general solution of the given differential equation.

OR

Find the particular solution of the differential equation $\frac{dx}{dy} + x\cot y = 2y + y^2 \cot y, (y \neq 0)$

given that x = 0 when y = $\frac{\pi}{2}$

20. Find the value of λ, if the four points A(-6, 3,2), B(3, λ, 4), C(5, 7, 3) and D(-13, 17, -1) are coplanar.

21. 7Find the vector and Cartesian equation of the plane passing through (-1, -1, 2) and perpendicular to the planes 3x + 2y – 3z = 1 and 5x – 4y + z = 5.

22. Bag I contains 1 white, 2 black and 3 red balls; Bag II contains 2 white, 1 black and 1 red balls; Bag III contains 4 white, 3 black and 2 red balls. A bag is chosen at random and two balls are drawn from it with replacement. They happen to be one white and one red. What is the probability that they came from Bag II.

23. Find the mean, variance of the numbers obtained on throwing a die having written 2 on one face, 3 on two faces, 5 on one face and 6 on two faces.

SECTION D

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24. Find the coordinates of the point where the line through the points (3, -4,-5) and (2, -3, 1) crosses the plane determined by the points A(1, 2, 3), B(2, 2, 1) and C(-1, 3, 6)

25. Evaluate $\int_{-2}^{3} (3x^2 - 2x + 4)dx$ as the limit of a sum.

OR

Evaluate $\int_{0}^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2 \sin^2 x}$

26. If A = $\begin{pmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{pmatrix}$, then find $A^{-1}$and hence solve the system of equations:

3x + 4y + 7z = 14, 2x – y + 3z = 4 and x + 2y – 3z = 0

OR

Find the inverse of $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{pmatrix}$ using elementary row transformations and hence solve the matrix equation XA = [1 0 1]

27. If the function $f : R \to R$ *be defined by* $f(x) = 2x - 3$ *and* $g : R \to R$ *by* $g(x) = x^3 + 5$, then find $f \circ g$ and show that $f \circ g$ is invertible. Also find $(f \circ g)^{-1}$ *and* $(f \circ g)^{-1}(9)$

OR

If the operation * on Q-{1}, defined by a*b = a + b – ab for all a, b $\in$ Q-{1}, then verify the following

a) Is * Commutative?

b) Is * associative?

c) Find the identity element

d) Find the inverse of 'a' for each a $\in$ Q-{1}

28. Find the area bounded by the curve $x^2 = 4y$, x – axis and the straight line x = 4y – 2.

29. A dietician wants to develop a special diet using two foods X and Y. Each packet (containing 30g) of food X contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Y contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. Make an LPP, to find how many packets of each food should be used to minimize the amount of vitamin A in the diet and solve it graphically.

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