Most Important -120

1. In a seminar, the number of participation in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required if in each room the same numbers of participants are to be seated and all of them being in the same subject.

2. If the HCF of 210 and 55 is expressible in the form $210 \times 5 + 55y$, find y.

3. Show that any positive odd integer is of the form $4q + 1$ or $4q + 3$, where q is some integer.

4. The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, find the other.

5. Prove that $\sqrt{2} + \sqrt{5}$ is irrational.

6. Find the zeros of the quadratic polynomial $f(x) = 6x^2 - 3$, and verify the relationship between the zeros and its coefficients:

7. If α and β are the zeros of the polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, find the value of k.

8. if α and β are the zeros of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, find a polynomial whose zeros are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.

9. Find all the zeros of the polynomial $2x^3 + x^2 - 6x - 3$, if two of its zeros are $-\sqrt{3}$ and $\sqrt{3}$.

10. Find the values of a and b so that $x^4 + x^3 + bx^2 + ax + b$ is divisible by $x^2 + 1$.

11. On dividing the polynomial $f(x) = x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient q(x) and remainder r(x) where $q(x) = x - 2$, $r(x) -2x + 4$ respectively. Find the polynomial g(x).

12. Draw the graphs of $2x + y = 6$ and $2x - y + 2 = 0$. Shade the region bounded by these lines and x-axis. Find the area of the Shaded region.

13. Solve the following system of equation (i)$8v - 3u = 5uv$; $6v - 5u = -2uv$ (ii) $217x + 131y = 913$; $131x + 217y = 827$

14. Solve: (i)$x + y = a + b$; $ax - by = a^2 - b^2$

15. Determine the values of m and n so that the following system of linear equations have infinite number of solutions:
$(2m - 1)x + 3y - 5 = 0$; $3x + (n-1)y - 2 = 0$

16. A and B each have certain number of oranges. A says to B, "if you give me 10 of your oranges, I will have twice the number Of oranges left with you. "B replies, "if you give me 10 of your oranges, I will have the same number of oranges as left with you. "Find the number of oranges with A and B separately.

17. The sum of a two-digit number and the number obtained by reversing the order of its digits is 165. If the digits differ by 3, Find the number.

18. **The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are Decreased by 6, then the denominator becomes 12 times the numerator. Determine the fraction.**

19. A boat covers 32km upstream and 36km downstream in 7 hours. Also, it covers 40km upstream and 48km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

20. A person invested some amount at the rate of 12% simple interest and some other amount at the rate of 10% simple interest. He received yearly interest of Rs. 130. But if he had interchanged the amounts invested, he would have received Rs. 4 more as interest. How much amount did he invest at different rates?

21. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.

22. Solve the following quadratic equations by factorization method: (i) $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

(ii) $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

23. Find the roots of the equation $a^2x^2 - 3abx + 2b^2 = 0$ by the method of completing the square.

24. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

25. If the equation $(1+ m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1 + m^2)$

26. O Girl! Out of a group of swans, 7/2 times the square root of the number are playing on the shore of a tank. The two remaining ones are playing, with amorous fight, in the water. What is the total number of swans?

27. In a flight of 600km, a aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200km/hr and the time of flight increased by 30 minutes. Find the duration of flight.

28. Seven years ago Varun's age was five times the square of Swati's age. Three years hence Swati's age will be two fifth of Varun's age. Find their present ages.

29. A chess board contains 64 equal squares and the area of each square is 6.25 cm^2. A border round the board is 2 cm wide. Find the length of the side of the chess board.

30. Two pipes running together can fill a tank in 100/9 minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

31. The angry Arjun carried some arrows for fighting with Bheeshm. With half the arrows, he cut down the arrows thrown by Bheeshm on him and with six other arrows he killed the rath driver of Bheeshm. With one arrow each he knocked down respectively the rath, flag and bow of Bheeshm. Finally, with one more than four times the square root of arrows he laid Bheeshm unconscious on an arrow bed. Find the total number of arrows Arjun had.

32. If the m^{th} term of an A.P. be 1/n and n^{th} term be 1/m, then show that its $(mn)^{th}$ term is 1

33. If m times the m^{th} term of an A.P. is equal to n times its n^{th} term, show that the $(m+n)^{th}$ term of A.P. is zero.

34. If 2x, x+10, 3x+2 are in A.P., Find the value of x.

35. Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers

36. The sum of n, 2n, 3n terms of an A.P. are S_1, S_2, S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

37. If in an A.P. the sum of m terms is equal to n and the sum of n terms is equal to m, then prove that the sum of (m + n) terms is – (m + n).

38. **150 workers were engaged to finish a piece of work in a certain numbers of days. Four workers dropped the second day, four more workers dropped the third day and so on. It takes 8 more days to finish the work now. Find the number of days in which the work was completed.**

39. The sum of the first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the 28^{th} term of this A.P.

40. IF a point A(0,2) is equidistant from the points B (3, p) and C (p, 5), then find the value of p.

41. Determine the ratio in which the line 3x + y – 9 = 0 divides the segment joining the points (1, 3) and (2, 7).

42. If x – 2y + k = 0 is a median of the triangle whose vertices are at points A (-1, 3), B(0, 4) and C (-5, 2) find the value of k.

43. If A (4, -6), B (3, -2) and C (5, 2) are the vertices of triangle ABC, then verify the fact that a median of a triangle ABC divides it into two triangles of equal areas.

44. If the points (p, q), (m, n) and (p – m, q – n) are collinear, show that pn = qm.

45. If the points P (-3, 9), Q (a, b) and R (4, -5) are collinear and a + b = 1, find the value of a and b.

46. Find the ratio in which the point P (x, 2) divides the line segment joining the points A (12, 5) and B (4, -3). Also find the value of x.

47. A bridge across a river makes an angle of 45^o with the river bank. If the length of the bridge across the river is 150m, what is the width of the river?

48. The angles of elevation of the top of a tower from two points at distances x and y metres from the base and in the same straight line with it are complementary. Prove that the height of the tower is $\sqrt{xy}$ metres.

49. At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is 5/12. On walking 192 metres towards the tower, the tangent of the angle of elevation is 3/4. Find the height of the tower.

50. The angle of elevation of a jet plane from a point A on the ground is 60^o. After a flight of 30 seconds, the angle of elevation changes to 30^o. If the jet plane is flying at a constant height of $3600\sqrt{3}$m, find the speed of the jet plane.

51. The angle of elevation of a cloud from a point 60 m above a lake is $30^o$and the angle of depression of the reflection of cloud in the lake is 60^o. Find the height of the cloud.

52. A round balloon of radius r subtends an angle α at the eye of the observer while the angle of elevation of its centre is β. Prove that the height of the centre of the balloon is **r sin β cosec α/2.**

53. From the top of a hill, the angles of depression of two consecutive kilometre stones due east are found to be 30^o and 45^o. Find the height of the hill.

54. Find the probability that a leap year selected at random will contain 53 Wednesday.

55. Three unbiased coins are tossed together. Find the probability of getting:
(i) all heads (ii) two heads (iii) one head (iv) at least two heads

56. A bag containing 5 red balls and some blue balls. If the probability of drawing a blue ball is thrice that of a red ball, find the number of blue balls in the bag.

57. One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability that the card drawn is:
(i) an ace (ii) red (iii) either red or king (iv) red and a king
(v) a face card (vi) a red face card (vii) '2' of spades (viii) '10' of a black suit

58. Cards numbered 1 to 30 are put in a bag. A card is drawn at random from this bag. Find the probability that the number on the drawn card is: (i) not divisible by 3 (ii) a prime number greater than 7 (iii) not a perfect square number.

59. **Divide a line segment of length 9 cm internally in the ratio 4:3.**

60. Construct a triangle PQR with side QR = 7 cm, PQ = 6 cm and $\angle PQR = 60^o$. Then construct another triangle whose sides are 3/5 of the corresponding sides of ΔPQR

61. Draw a pair of tangents to a circle of radius 4.5 cm, which are inclined to each other at an angle of 45^o

62. Draw a right triangle ABC in which AB= 6 cm, BC = 8 cm and $\angle B = 90^o$. Draw BD perpendicular from B on AC and draw a circle passing through the points B, C and D. Construct tangents from A to this circle.

63. A circle with centre O is inscribed in a quadrilateral ABCD such that, it touches sides BC, AD and CD at points P, Q, R and S respectively. If AB = 29 cm, AD = 23 cm, $\angle B = 90^o$ and DS = 5 cm, then find the radius of the circle.

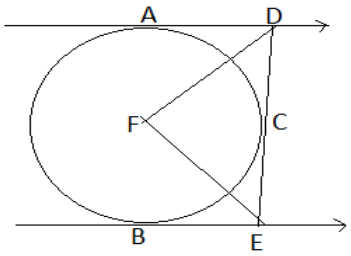
64. The sides AB, BC and CA of triangle ABC, touch a circle at P, Q and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11 cm, then find the length of BC.

65. A triangle PQR is drawn to circumscribe a circle of radius 8 cm such that the segments QT and TR, into which QR is divided by the point of contact T, are of length 14 cm and 16 cm respectively. If area of ΔPQR is 336 cm^2, find the sides PQ and PR.

66. AB is a chord of length 16 cm of a circle of radius 10 cm. The tangents at A and B intersect at a point P. Find the length of PA.

67. The radii of two concentric circles are 26 cm and 16 cm. AB is a diameter of the bigger circle. BD is tangent to the smaller circle touching it at D. Find the length AD.

68. In fig., l and m are two parallel tangents at A and B. The tangent at C makes an intercept DE between l and m. Prove that $\angle DFE = 90^o$



69. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$

70. The radius of the incircle of a triangle is 4 cm and the segments into which one side is divided by the point of contact are 6 cm and 8 cm. Determine the other two sides of the triangle.

71. If all the side of a parallelogram touch a circle, show that the parallelogram is a rhombus.

72. The sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P, Q and R respectively.
(i) AB + CQ = AC + BQ (ii) Area (ΔABC) = ½ (Perimeter of ΔABC) x r

73. A circle is touching the sides BC of ΔABC at P and touching AB and AC produced at Q and R respectively. Prove that : AQ = AR = ½ (Perimeter of ΔABC)

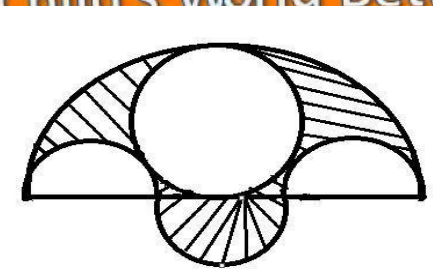
74. Two circles touch externally. The sum of their areas is 130 π sq. cm. and the distance between their centres is 14 cm. Find the radii of the circles.

75. A wheel has diameter 84 cm. Find how many complete revolutions must if take to cover 792 meters.

76. The minute hand of a clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 9 A.M. and 9:35 A.M.

77. In a circle of radius 21 cm, an arc subtends an angle of 60^o at the centre. Find
(i) the length of the arc (ii) area of the sector formed by the arc. (use π = 22/7)

78. . Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



79. Three cubes each of sides 5 cm are joined end to end. Find the surface area of the resulting cuboid.

80. A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel?

81. Water is flowing at the rate of 7 metres per second through a circular pipe whose internal diameter is 2 cm into a cylindrical tank the radius of whose base is 40 cm. Determine the increase in the water level in ½ hour.

82. Water is flowing at the rate of 3 km/hr through a circular pipe of 20 cm internal diameter into a circular cistern of diameter 10 m and depth 2 m. In how much time will the cistern be filled?

83. A cylindrical tank full of water is emptied by a pipe at the rate of 225 litres per minute. How much time will it take to empty half the tank, if the diameter of its base is 3 m and its height 3.5 m? [Use $\pi = 22/7$]

84. A right triangle, whose sides are 15 cm and 20 cm, is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. [Use $\pi = 22/7$]

85. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2

86. A hemisphere depression is cut-out from one face of the cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface are of the remaining solid.

87. A bucket is in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.

88. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is 8/9 of the curved surface of the whole cone, find the ratio of the line-segment into which the cone's altitude is divided by the plane.

89. The height of a right circular cone is trisected by two planes drawn parallel to the base. Show that the volumes of the three portions starting from the top are in the ratio 1:7:19.

90. A shuttle cock used for playing badminton has the shape of a frustum of a cone mounted on a hemisphere. The external diameters of the frustum are 5 cm and 2 cm, the height of the entire shuttle cock is 7 cm. Find its external surface area.

91. A bucket made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with diameters of its lower and upper ends as 16 cm and 40 cm respectively. Find the volume of the bucket. Also, find the cost of the bucket if the cost of metal sheet used is Rs 20 per 100 cm^2. [Use $\pi = 3.14$]

92. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Show that their volumes are in the ratio 1:2:3

93. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

94. Prove that the lengths of tangents drawn from an external point to a circle are equal.

95. **Prove that if a line drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio**.

96. In ΔABC, LM || AB. If AL = x -3, AC = 2x, BM = x -2 and BC = 2x + 3, find the value of x.

97. D is a point on the side BC of ΔABC such that $\angle ADC = \angle BAC$. Prove that $CA^2 = CB \times CD$

98. Prove that the ratio of the areas of two similar triangles are equal to the ratio of the squares of any two corresponding sides.

99. Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.

100. P and Q are mid-points of the sides CA and CB respectively of a ΔABC, right angled at C. Prove that:
(i) $4\,AQ^2 = 4\,AC^2 + BC^2$ (ii) $4\,BP^2 = 4\,BC^2 + AC^2$ (iii) $4\,(AQ^2 + BP^2) = 5\,AB^2$

101. The perpendicular AD on the base BC of a ΔABC intersects BC at D so that DB = 3CD. Prove that $2AB^2 = 2AC^2 + BC^2$

102. In an equilateral ΔABC the sides BC is trisected at D. Prove that $9\ AD^2 = 7\ AB^2$

103. ABC is a right-angled triangle right angled at A. A circle is inscribed in it the lengths of the two sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle.

104. Prove that in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

105. In ΔOPQ right angled at P, OP = 7 cm, OQ – PQ = 1 cm. Determine the values of Sin Q and Cos Q.

106. Show that: (i) $2(\cos^2 45^o + \tan^2 60^o) - 6(\sin^2 45^o - \tan^2 30^o) = 6$ (ii) $2(\cos^4 60^o + \sin^4 30^o) - (\tan 260^o + \cot^2 45^o) + 3\sec^2 30^o = 1/4$

107. If $\text{Sin}(A + B) = 1$ and $\text{Cos}(A - B) = \sqrt{3}/2$, $O^o < A + B \leq 90^o$, A > B then find A and B.

108. Prove that: (i) $\tan 1^o \tan 2^o \tan 3^o \ldots \tan 89^o = 1$ (ii) $\text{Cos}1^o\ \text{Cos}2^o\ \text{Cos}3^o \ldots \text{Cos}180^o = 0$

109. If Sec 4A = Cosec $(A - 20^o)$, where 4A is an acute angle, find the value of A.

110. Prove that: (i) $(\text{Sin}\ \theta + \text{Cosec}\ \theta)^2 + (\text{Cos}\ \theta + \text{Sec}\ \theta)^2 = 7\ \tan^2\theta\ + \cot^2\theta$ (ii) $\text{Sec}^4\theta - \text{Sec}^2\theta = \tan^2\theta + \tan^4\theta$

111. If $\text{Cos}\ \theta + \text{Sin}\ \theta = \sqrt{2}\ \text{Cos}\ \theta$, show that $\text{Cos}\ \theta - \text{Sin}\ \theta = \sqrt{2}\ \text{Sin}\ \theta$

112. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$

113. **If $\text{Cos}\ \theta + \text{Cos}^2\ \theta = 1$, prove that: $\text{Sin}^{12}\theta + 3\ \text{Sin}^{10}\theta + 3\text{Sin}^8\theta + \text{Sin}^6\theta + 2\text{Sin}^4\theta + 2\text{Sin}^2\theta - 2 = 1$**

114. If $\text{Sec}\ \theta = x + 1/4x$, prove that: $\sec\theta + \tan\theta = 2x$ or, $1/2x$

115. Find the value of p, if the mean of the following distribution is 7.5.

X :	**3**	**5**	**7**	**9**	**11**	**13**
Y :	**6**	**8**	**15**	**P**	**8**	**4**

116. If the median of the following frequency distribution is 46, find the missing frequencies.

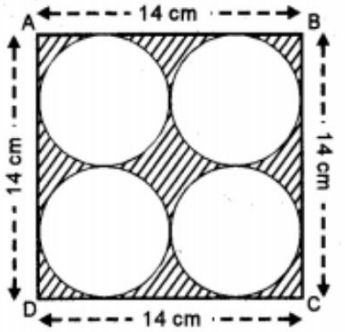
Variable :	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	**60 – 70**	**70 – 80**	**Total**
Frequency :	**12**	**30**	**?**	**65**	**?**	**25**	**18**	**229**

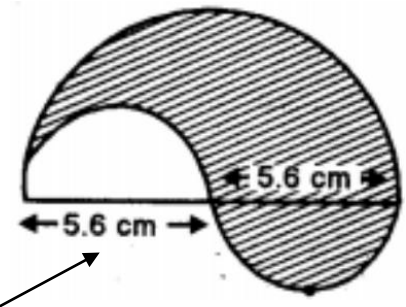
117. For the following grouped frequency distribution find the mode :

Class :	**3 – 6**	**6 – 9**	**9 – 12**	**12 - 15**	**15 – 18**	**18 – 21**	**21 - 24**
Frequency :	**2**	**5**	**10**	**23**	**21**	**12**	**3**

118. Prove that: (i) $(1 + \cot\theta - \text{cosec}\ \theta)(1 + \tan\theta + \sec\theta) = 2$ (iv) $\text{Sin}^6 A + \text{Cos}^6 A = 1 - 3\text{Sin}^2 A\ \text{Cos}^2 A$

119. Find the area of the shaded region in figure , where ABCD is a square of side 14 cm [Use $\pi = 22/7$]





120. Calculate the area of the shaded region in figure.