Class X: Chapter 4: Quadratic Equation
Assignment: Solution of Quadratic Equation

Quadratic Equations

1. The general form of a quadratic equations is $ax^2 + bx + c = 0$, $a \neq 0$, ab, b, a and c are real numbers.

2. A real number α is said to be a root of the quadratic equations $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

3. **Discriminant**
   The expression $b^2 - 4ac$ is called discriminant of the equation $ax^2 + bx + c = 0$ and is usually denoted by D.
   Thus discriminant $D = b^2 - 4ac$

4. The quadratic equation $ax^2 + bx + c = 0$ has real solutions only if D is positive

   Quadratic Formula

   The roots of the equation $ax^2 + bx + c = 0$ are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$.

5. **Nature of Roots**
   A quadratic equation $ax^2 + bx + c = 0$ has
   (i) two distinct real roots if $D > 0$
   (ii) two equal (i.e., coincident) roots if $D = 0$ and
   (iii) no real roots if $D < 0$.

6. Check whether the following are quadratic equations:
   (i) $(x - 2)^2 + 1 = 2x - 3$
   (ii) $x(x + 1) + 8 = (x + 2)(x - 2)$
   (iii) $x(2x + 3) = x^2 + 1$
   (iv) $(x + 2)^3 = x^3 - 4$
   (v) $(x + 1)^2 = 2(x - 3)$
   (vi) $x^2 - 2x = (-2)(3 - x)$
   (vii) $(x - 2)(x + 1) = (x - 1)(x + 3)$
   (viii) $(x - 3)(2x + 1) = x(x + 5)$
   (ix) $(2x - 1)(x - 3) = (x + 5)(x - 1)$
   (x) $x^2 + 3x + 1 = (x - 2)^2$
   (xi) $(x + 2)^3 = 2x(x^2 - 1)$
   (xii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

7. Is $x = -2$ a solution of the equation $x^2 - 2x + 8 = 0$?

8. Show that $x = -3$ is a solution of the equation $x^2 + 6x + 9 = 0$.

9. Which of the following are the roots of $3x^2 + 2x - 1 = 0$?

(i) -1 (ii) $\frac{1}{3}$ (iii) $-\frac{1}{2}$

10. In each of the following cases find whether the given values are the solution of the given quadratic equation.

(i) $x^2 + 6x + 5 = 0$ $(x = 1$ and $x = 5)$

(ii) $x^2 + x + 1 = 0$ $(x = 1$ and $x = -1)$

(iii) $9x^2 - 3x - 2 = 0$ $(x = \frac{-1}{3}$ and $x = \frac{2}{3})$

(iii) $x^2 + \sqrt{3}x - 6 = 0$ $(x = \sqrt{3}$ and $x = -\sqrt{3})$

11. (i) Find the value of k for which x = 1 is a root of the equation $x^2 + kx + 3 = 0$.

(ii) Find the value of m for which $x = \frac{2}{3}$ is a root of the equation $mx^2 - x - 2 = 0$

Solve by factorization

Find the roots of the following quadratic equations by factorisation:

1. $x^2 - 3x - 10 = 0$
2. $2x^2 + x - 6 = 0$
3. $6x^2 - x - 2 = 0.$
4. $2x^2 - 5x + 3 = 0$
5. $x^2 + 2x - 8 = 0$
6. $2x^2 + x + \frac{1}{8} = 0$
7. $100x^2 - 20x + 1 = 0$
8. $3x^2 - 2\sqrt{6}x + 2 = 0$.
9. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
10. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$
11. $x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$
12. $\sqrt{6}x^2 + \sqrt{2}x - 5\sqrt{3}x - 5 = 0$
13. $p^2x^2 + (p^2 - q^2)x - q^2 = 0$
14. $36x^2 - 12ax + a^2 - b^2 = 0$
15. $x^2 - 2ax + a^2 - b^2 = 0$
16. $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$
17. $a^2b^2x^2 + b^2x - a^2x - 1 = 0$
18. $a(x^2 + 1) - x(a^2 + 1) = 0$
19. $x^2 - x - a(a + 1) = 0$
20. $x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$

21. $abx^2 + (b^2 - ac)\,x - bc = 0$

22. $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}, x \neq 2, 4$

By the method of completing squares, solve each of the following equations

23. $x^2 + 2x - 8 = 0$
24. $5x^2 - 2x - 2 = 0$
26. $2x^2 + x - 4 = 0$
27. $5x^2 - 6x - 2 = 0$
28. $4x^2 + 3x + 5 = 0$
29. $2x^2 - 7x + 3 = 0$
30, $2x^2 + x - 4 = 0$
31. $x^2 + 4\sqrt{3}\,x + 3 = 0$
32. $2x^2 + x + 4 = 0$

Using quadratic formula or otherwise, solve each of the following equations

33. $3x^2 - 5x + 2 = 0$
34. $x^2 + 4x + 5 = 0$
35. $2x^2 - 2\sqrt{2}\,x + 1 = 0$
36. $2x^2 - 3x + 5 = 0$
37. $3x^2 - 4\sqrt{3}\,x + 4 = 0$
38. $2x^2 - 6x + 3 = 0$

39. Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.

Using quadratic formula, solve the following equation for x :

40. $abx^2 + (b^2 - ac)x - bc = 0$
41. $3x^2 + 9x + 4 = 0$
42. $3y^2 - 32y + 12 = 0$
43. $6a^2x^2 - 7abx - 3b^2 = 0$
44. $10x - \frac{1}{x} = 3,\ x \neq 0$
45. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$
46. $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

Solve:

47. $\frac{1}{a+b+c} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}; a \neq 0,\ b \neq 0, x \neq 0$
48. $(a^2 - b^2)x^2 - 2(a^2 + b^2)x + a^2 - b^2 = 0$
49. $acx^2 + (2bc - a^2)x - 2ab = 0$

50. $(a+b)^2x^2 + 8(a^2 - b^2)x + 16(a-b)^2 = 0,\ a+b \neq 0,\ a \neq b$

51. $\left(\frac{x-2}{x+2}\right)^2 - 4\left(\frac{x-2}{x+2}\right) + 3 = 0,\ \ x \neq -2$

52. $\left(\frac{2x}{x-5}\right)^2 + \frac{10x}{x-5} - 24 = 0,\ \ x \neq 5$

53. $x + \frac{1}{x} = 3,\ x \neq 0$

54. $\frac{1}{x} - \frac{1}{x-2} = 3,\ x \neq 0,\ 2$

55. $x - \frac{1}{x} = 3,\ x \neq 0$

56. $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30},\ x \neq -4,\ 7$

57. $\frac{2}{x^2} - \frac{5}{x} + 2 = 0,\ x \neq 0$

58. $10ax^2 - 6x + 15ax - 9 = 0,\ a \neq 0$

59. $\frac{x+3}{x+2} = \frac{3x-7}{2x-3},\ \ x \neq -2,\ x \neq \frac{3}{2}$

60. $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x},\ x \neq 0, 1,\ 2$

61. $\frac{x+1}{x-1} + \frac{x-1}{x+1} = \frac{5}{6},\ \ x \neq 1, x \neq -1$

62. $a^2x^2 - 3abx + 2b^2 = 0$ **[Ans.x = 2b/a, x = b/a]**

63. $10ax^2 - 6x + 15ax - 9 = 0,\ a \neq 0$ **[Ans. x = 3/5a, -3/2]**

64. Solve $p / (px - 1) + q / (qx - 1) = p + q$

Extra

65. Solve $(x + 1)(x + 2)(x + 3)(x + 4) = 120$

66. Solve $(x + y)^2 - 2(x + y) = 15,\ xy = 6$

67. $\left(\frac{x}{x+1}\right)^2 - 5\left(\frac{x}{x+1}\right) + 6 = 0,\ \ x \neq -1$. $\left\{-2, \frac{-3}{2}\right\}$

68. $\left(\frac{2x-3}{x-1}\right) - 4\left(\frac{x-1}{2x-3}\right) = 3$ given that $x \neq 1$ and $x \neq \frac{3}{2}$ $\left\{\frac{1}{2}, \frac{4}{3}\right\}$

69. $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5$, given that $x \neq$ -3 and $x \neq \frac{1}{2}$ $\left\{-10, \frac{-1}{5}\right\}$

70. $4^{(x+1)} + 4^{(1-x)} = 10$ $\left\{\frac{1}{2}, \frac{-1}{2}\right\}$

71. $(x^2 - 5x)^2 - 7(x^2 - 5x) + 6 = 0$. $\left\{6, -1, \frac{(5+\sqrt{29})}{2}, \frac{(5-\sqrt{29})}{2}\right\}$

72. $(x^2 + 3x + 2)^2 - 8(x^2 + 3x) - 4 = 0$. {- 4, - 3, 0, 1}

73. $x^{2/3} + x^{1/3} - 2 = 0$ {- 8, 1}

74. $3\left(x^2+\frac{1}{x^2}\right)-16\left(x+\frac{1}{x}\right)+26=0 \quad x \neq 0$ $\left\{1, 3, \frac{1}{3}\right\}$

75. $4\left(x^2+\frac{1}{x^2}\right)-8\left(x+\frac{1}{x}\right)+3=0, \ x \neq 0$ $\left\{2, \frac{1}{2}\right\}$

76. $\left(x^2+\frac{1}{x^2}\right)-3\left(x-\frac{1}{x}\right)-2=0, \ x \neq 0$ $\left\{1, -1, \frac{(3 \pm \sqrt{13})}{2}\right\}$

77. $6\left(x^2+\frac{1}{x^2}\right)-25\left(x-\frac{1}{x}\right)+12=0, \ x \neq 0$. $\left\{2, \frac{-1}{2}, 3, \frac{-1}{3}\right\}$

78. $\left(x+\frac{1}{x}\right)^2-\frac{3}{2}\left(x-\frac{1}{x}\right)-4=0, \ x \neq 0$ $\left\{1, -1, 2, \frac{-1}{2}\right\}$

79. $4\left(x-\frac{1}{x}\right)^2+8\left(x+\frac{1}{x}\right)=29, \ x \neq 0$. $\left\{2, \frac{1}{2}, \frac{(-9+\sqrt{65})}{4}, \frac{(-9-\sqrt{65})}{4}\right\}$

80. $6\sqrt{\frac{x}{x+4}}-2\sqrt{\frac{x+4}{x}}=11$ $\left\{\frac{-16}{3}\right\}$

81. The value of $\sqrt{6+\sqrt{6+\sqrt{6+}}}$..... is ?

Nature Of Roots

82. Examine, whether the following equations have real roots :
    (i) $x^2 + x + 1 = 0$
    (ii) $3x^2 + 2x - 1 = 0$
    (iii) $2x^2 + 5x + 5 = 0$
    (iv) $x^2 + 4x + 4 = 0$
    (iv) $2x^2 + x - 1 = 0$
    (vi) $x^2 + 5x + 5 = 0$

83. Find the values of k for each of the following quadratic equations, so that they have two real equal roots.
    (i) $2x^2 + kx + 3 = 0$
    (ii) $kx(x - 2) + 6 = 0$
    (iii) $5x^2 - 4x + 2 + k(4x^2 - 2x - 1) = 0$
    (iv) $(4 - k)x^2 - (2k + 4)x + (8k + 1) = 0$

84. For what value of p, each of the following equations has real roots
(i) $px^2 + 6x + 1 = 0$
(ii) $4x^2 + 8x - p = 0$
(iii) $2x^2 + px + 18 = 0$

85. If the equation $(1 + m^2)\ x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2\ (1 + m^2)$.

86. If the roots of the equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $b + c = 2a$.

Word Problem

87. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.

88. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

89. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.

90. John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

91. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. We would like to find out the number of toys produced on that day.

92. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

93. The product of two consecutive positive integers is 306. We need to find the integers.

94. Find two numbers whose sum is 27 and product is 182.

95. Find two consecutive positive integers, sum of whose squares is 365.

96. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

97. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

98 Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

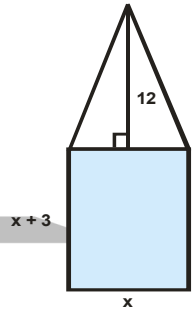
99. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

95. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

96. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

97. The area of a rectangular plot is 528 m2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

98. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m (see Fig.). Find its length and breadth.



99. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

100. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

101. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

102. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

103. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

104. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.

105. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

106. In a cricket match Kumble took three wickets less than twice the number of wickets taken by Shrinath. The product of the number of wickets taken by these two is 20, find the number of wickets than taken by each.

107. The numerator of a fraction is 3 less than its denominator. If 1 is added to the denominator, the fraction is decreased by $\frac{1}{15}$. Find the fraction.

107. Out of a number of some Saras birds, one-fourth the numbers are moving about in lotus plants, $\frac{1}{9}^{th}$ as well as 7 times the square root of the number move on a hill and 56 birds remain in Vakula trees. Find the total number of birds.

108. On a pillar of 9 cubits high is perched a peacock. From a distance of 27 cubits a snake is coming to its hole at the bottom of the pillar. Seeing the snake, the peacock pounces upon it. If their speeds are equal, at what distance from the hole is the snake caught?

109. A swimming pool is filled with thee pipes with uniform flow. The first two pipes operating simultaneously, fill the pool in the same time during which the pool is filled by the third pipe alone. The second pipe fills the pool five hours faster than the first pipe and four hours slower than the third pipe. Find the time required by each pipe to fill the pool separately.

110. Some students planned a picnic. The budget for food was Rs. 480. But eight of them failed to go and thus the cost of food for each member increased by Rs. 10. How many students attended the picnic?

111. 7 years ago age of Varun was five times the square of the age of Swati. After 3 years, age of Swati will be $\frac{2}{5}$ of the age of Varun. Find their present ages.

112. A two digit number is such that the product of its digits is 35. When 18 is added to the number, the digits inter-change their places. Find the number.

113. A two digit number is seven times the sum of its digits and is also equal to 12 less than three times the product of its digits. Find the number.

114. 300 apples are distributed equally among a certain number of students. Had there been 10 more students, each would have received one apple less. Find the number of students.

115. If the list price of a book is reduced by Rs. 5, a person can buy 5 more books for Rs. 300. Find the original list price of the book.

116. Rs. 9000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got Rs. 160 less. Find the original number of persons.

117. Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill the cistern, find the time taken by each pipe to fill the cistern.

118. The angry Arjun carried some arrows for fighting with Bheeshm. With half the arrows he cut down the arrows thrown by Bheeshm on him and with six other arrows he killed the charioteer of Bheeshm With one arrow each he knowked down respectively the rath, flag and bow of Bheeshm. Finally with one more than four times the square root of arrows he laid Bheeshm unconscious on an arrow-bed. Find the total number arrows Arjun had.

119. Abhishek takes 6 days less than the time taken by Anubhav to finish a piece of work. If both of them together finish the work in 4 days, find the time taken by Anubhav alone to finish the work.

120. To cover 14 km distance, Anubhav cycles at a certain average speed. If he reduces his average speed by 1 km/h, he takes 20 minutes more to cover the same distance. Find the original speed.

121. Two trains leave a railway station at the same time. The first train travels due West and the second due North. The first train travels 5 km/h faster than the second. If after two hours they are 50 km apart, find the average speed of each train.

122. One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of the river. Find the total number camels.

123. A factory kept increasing its output by the same percentage every year. Find the percentage increase if it is known that the output is doubled in the last 2 years.

124. A fisherman can row a boat 8 km downstream and return in 100 minutes. If the speed of the stream is 2 km per hour, find the speed of the boat in still water.

125. A line segment AB of 2 m length is divided at C into two parts such that $AC^2 = AB \times CB$. Find the length of CB.

126. A two digit number is four times the sum and three times the product of its digits. Find the number.

127. One year ago a man was 8 times as old as his son. Now his age is equal to the square of his son's age. Find their present ages.

128. A dealer sells an article for Rs. 75 and gains as much percent as the cost price of the article. Find the cost price of the article.

129. A swimming pool is filled with three pipes with uniform flow. The first two pipes operating simultaneously, fill the pool in the same time during which the pool is filled by the third pipe alone. The second pipe fills the pool five hours faster than the first

pipe and four hours slower than the third pipe. Find the time required by each pipe to fill the pool separately.

130. A chess board contains 64 equal squares and the area of each square is 6.25 cm2. A border round the board is 2 cm width. Find the length of the side of the chess board.

131. A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work.

132. A farmer wishes to grow a 100 m2-rectangular vegetable garden. Since he has with the only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of his house act as the fourth side fence. Find the dimensions of his garden.

133. In a flight of 600 km, a aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the duration of flight.

134. Ashu is x years old while his mother Mrs. Veena is x^2 years old. Five years hence Mrs. Veena will be three times old as Ashu. Find the their present ages.

135. A piece of cloth costs Rs. 35. If the piece were 4 m longer and each meter costs Rs. one less, the cost would remain unchanged. How long is the piece?

136. The population of a village is 5000. If in a year, the number of males were to increase by 5% and that of a female by 3% annually, the population would grow to 5202 at the end of a year. Find the number of males and females in the village at present.