MATHEMATICS
CLASS XII

Time: 3 hours **MM: 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of **29** questions divided into three sections **A**, **B,C** and **D**. Section **A** comprises **4** questions of **one mark** each, Section **B** comprises **8** questions of **two marks** each , Section **C** comprises 13 questions of **four marks** each and Section **D** comprises 6 questions of **six marks** each.*
3. *All questions in Section **A** are to be answered in one word, one sentence or as per the exact requirement of the questions.*
4. *Use of calculator is not permitted. You may ask for logarithmic tables, if required.*

SECTION – A

Q1 Let $f: R \to R$ be defined as f(x) = 10x + 7 . Find the function g : R $\to R$ such that g o f = f o g = 1_R . 1

Q2 If A = $\begin{bmatrix} 46 & 69 & 71 \\ 102 & 501 & 28 \\ 39 & 38 & 40 \end{bmatrix}$ and |5A| = |K|A|, find k . 1

Q3 Let $\vec{a}$ *and* $\vec{b}$ be two unit vectors and θ is the angle between them. If $\vec{a}+\vec{b}$ is a unit vector then find θ. 1

Q4 If the binary operation * on the set Z of integers is defined by a* b = a + b – 5 , then write the identity elements for the operation * in Z. 1

SECTION – B

Q5 Write the value of $\sin\left(2\sin^{-1}\left(\frac{3}{5}\right)\right)$. 2

Q6 Find X and Y , if 2

(i) 2X +3 Y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X+2Y=\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

Q7 Differenitate the following function with respect to x : f(x) = $\tan^{-1}\left(\frac{1-x}{1+x}\right)-\tan^{-1}\left(\frac{x+2}{1-2x}\right)$. 2

Q8 If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, find the approximate error in calculating its surface area. 2

Q9 $Evaluate: \int e^x \frac{x^2+1}{(x+1)^2}dx$. 2

Q10 Form a differential equation representing the given family of curve by eliminating arbitrary constants a and b. 2

$y=e^x(a\cos x+b\sin x)$

Q11 If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k} and \vec{c}=3\hat{i}+\hat{j}$ are such that $\vec{a}+\lambda 2\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ . 2

Q12 A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. 2

Section – C

Q13 Two cricket teams honoured their players for three values excellent bowling , to the point bowling and unparallel fielding by giving Rs x, Rs y and Rs z per player respectively. The first team paid 2,2 and 1 player respectively for the above values with a total prize money of Rs 11 lakhs, while the second team paid 1,2 and 2 players respectively for these values with a total prize money of Rs 9 lakhs. If the total award money for one person each for these values amount to Rs 6 lakhs, then express the above situation as a matrix equation. 4

Is it possible to find the award money per person for each value?

Q14 4

Let $f(x)=\begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & if \quad x<\pi/2 \\ a \quad , & if \ x=\pi/2 \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & ,if \quad x>\pi/2 \end{cases}$

If f(x) be a continuous function at x = $\pi/2$, find a and b.

Q15 4

$f(x)=\begin{cases} 2x+3, & x\le 1 \\ \dfrac{x^2-4x+20}{4}, & x>1 \end{cases}$ Check the differentiability of f (x) at x = 1 .

Q16 Find the equation of the normal to the curve $x^2=4y$ which passes through the point (1,2). 4

Q17 A magazine seller has 500 subscribes and collects annual subscription charges of Rs . 300 per subscriber. She proposes to increases the annual subscription charges and it is believed that for every increase of Rs 1, one subscriber will discontinue. What increase will bring maximum income to her? Make appropriate assumptions in order to apply derivatives to reach the solution. Write one important role of magazines in our lives. 4

Q18 Evaluate : $\int x^2 \tan^{-1} x dx$ 4

OR

Evaluate : $\int \sqrt{\dfrac{1+x}{1-x}}dx, x\ne 1.$

Q19 Solve : $x\dfrac{dy}{dx}+y-x+xy\cot x=0\left(x\ne 0\right)$ 4

Q20 Show that four points A,B,C and D whose position vectors are 4

$4\hat{i}+5\hat{j}+\hat{k},-\hat{j}-\hat{k},3\hat{i}+9\hat{j}+4\hat{k}$ and $4\left(-\hat{i}+\hat{j}+\hat{k}\right)$ respectively are coplanar.

OR

Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{i}+2\hat{j}+\hat{k}$ *and* $-\hat{i}+3\hat{j}+4\hat{k}.$

Q21 Find the values of a so that the following lines are skew: 4

$\dfrac{x-1}{2}=\dfrac{y-2}{3}=\dfrac{z-a}{4},\dfrac{x-4}{5}=\dfrac{y-1}{2}=z.$

OR

Find the co – ordinates of the foot of perpendicular drawn from the point A (1,8,4) to the line joining B (0, -1, 3) and C (2, -3, -1).

Q22 In a hostel , 60% of the students read Hindi news paper, 40% read English news paper and 20% read both Hindi and English newspapers. A student is selected at random. 4

a) Find the probability that she reads neither Hindi nor English news papers.

b) If she reads Hindi newspaper, find the probability that she reads English news paper.
c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Q23 Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained . Find the probability distribution of the random variable X, and hence find the mean of the distribution. 4

Section – D

Q24 Let $f:[0,\infty)\to R$ be a function defined by f(x) = $9x^2 + 6x - 5$. Prove that f is not invertible. Modify, only the codomain of f to make f invertible and then find its inverse. 6

OR

Let * be a binary operation defined on $Q\times Q by (a,b)*(c,d)=(ac,b+ad)$, where Q is the set of rational numbers. Determine, whether * is commutative and associative. Find the identity element for * and the invertible elements of $Q\times Q$.

Q25 Using properties of determinants , prove that $\begin{vmatrix} \frac{(a+b)^2}{c} & c & c \\ a & \frac{(b+c)^2}{a} & a \\ b & b & \frac{(c+a)^2}{b} \end{vmatrix} = 2(a+b+c)^3$. 6

OR

If $p\neq 0, q\neq 0 and \begin{vmatrix} p & q & p\alpha+q \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$, then using properties of determinants , prove that at least one of the following statement is true : (a) p,q,r are in G.P., (b) α is a root of the equation $px^2 + 2qx + r = 0$.

Q26 Using integration, find the area of the triangle ABC with vertices as $A(-1,0)$, B(1,3) andC(3,2). 6

Q27 Evaluate : $\int_0^{\pi} \frac{x}{1+\sin x}dx$ 6

OR

Evaluate : $\int_1^3 \left[|x-1|+|x-2|+|x-3|\right]dx$

Q28 If lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then find the value of k and hence find the equation of the plane containing these lines. 6

Q29 One kind of cake required 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1 kg of fat, assuming that there is no shortage of other ingradients used in making the cakes. Formulate the above as a linear programming problem and solve graphically. 6

Answers

1 $g(y)=\frac{y-7}{10}$ 2 k = 5^3 3 $\theta=\frac{2\pi}{3}$ 4 e = 5 5 $\frac{24}{25}$

6 $X=\begin{bmatrix}\frac{2}{5} & \frac{-12}{5}\\ \frac{-11}{5} & 3\end{bmatrix}, \mathrm{Y}=\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\ \frac{14}{5} & -2\end{bmatrix}$ 7 $f'(x)=-\frac{2}{1+x^2}$ 8 $2.16\pi cm^2$ 9 $e^x-2e^x.\frac{1}{x+1}+C$

10 y" – 2y' + 2y = 0 11 8 12 3/8 13 $\begin{bmatrix}2&2&1\\1&2&2\\1&1&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1100000\\900000\\600000\end{bmatrix}, yes\ as \begin{vmatrix}2&2&1\\1&2&2\\1&1&1\end{vmatrix}\neq 0$

14 a = ½ , b = 4 16 x + y = 3 17 x = 100

18 $\frac{x^3\tan^{-1}x}{3}-\frac{x^2}{6}+\frac{1}{6}\log|x^2+1|+C$ OR $I=\sin^{-1}x-\sqrt{1-x^2}+C$ 19 $y=\frac{1}{x}-\cot x+\frac{C}{x\,\mathrm{sinx}}$

20 or $\pm 10\left(\hat{i}-\hat{j}+\hat{k}\right)$ 21 or $\left(\frac{-5}{3},\frac{2}{3},\frac{19}{3}\right)$ 22 a) 1/5 b) 1/3 c) ½ 23 $4\frac{2}{3}$

24 or $(a,b)\in Q\times Q, a\neq 0 is\left(\frac{1}{a},\frac{-b}{a}\right)$

26 4 sq . units 27 $\pi[1-0]=\pi$ or 5 28 5x – 2y – z – 6 = 0 29