CLASS XII
SAMPLE PAPER
MATHS

TIME: 3 Hrs **Max.Marks:100**

SECTION A

1. Let * be the binary operation defined on N defined by a*b = H.C.F. of a and b. Find the identity element if it exists.
2. Prove that $\sec^2(\tan^{-1} 2) + \text{cosec}^2(\cot^{-1} 3) = 15$.
3. If the matrix $\begin{pmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{pmatrix}$ .is non-invertible, find the value of a.
4. If |adj A| = 25 for a square matrix of order 3, find |4A|.
5. Find the direction cosines of a line that makes equal angle with the coordinate axes.
6. Find value of λ such that the line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to the plane 3x-y-2z=7.
7. Evaluate: $\int \frac{2^x}{\sqrt{1+4^x}} dx$.
8. 8. Evaluate: $\int_1^3 |2-x| dx$.
9. If $\vec{r}$ is any vector in space show that $\vec{r} = (\vec{r} \bullet \hat{i})\hat{i} + (\vec{r} \bullet \hat{j})\hat{j} + (\vec{r} \bullet \hat{k})\hat{k}$
10. If A is 3x4 matrix and B is a matrix such that A^TB and BA^T are both defined find the order of the matrix B.

SECTION B

11. Find the value of x if $\cot^{-1}\left\{2\tan\left(\cos^{-1}\frac{5}{13}\right)\right\} + \tan^{-1}\left\{2\tan\left(\sin^{-1}\frac{5}{13}\right)\right\}\frac{1}{5} = \tan^{-1} 4x$
12. If R→R, and a, b, c, d ϵ R such that (a,b) *(c,d) = (ac, b + ad) determine if * is commutative, associative. Find the identity element of the function

13. Using properties of determinants prove that $\begin{vmatrix} 3a & -a+b & -a+c \\ a-b & 3b & c-b \\ a-c & b-c & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$.

14. If $y = \cos^{-1}\left[\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right]$, find $\frac{dy}{dx}$

15. 15. $\int x\left(\frac{\sec 2x - 1}{\sec 2x + 1}\right)dx$

16. Find the intervals in which the function f(x) = $x^4 - 2x^2$, is Increasing or Decreasing. OR A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of the water in the tank is 4 m.

17. Evaluate: $\int \frac{\sin x}{\sin 4x} dx$ OR $\int \frac{1}{\sin x - \sin 2x} dx$

18. A and B throw a pair of die turn by turn. The first to throw 10 is awarded a prize. If B starts the game. What is the probability that A is getting prize.

19. Find whether the lines $\vec{r} = \hat{i} - \hat{j} - \hat{k} + \lambda\left(2\hat{i} + \hat{j}\right)$ and $\vec{r} = 2\hat{i} - \hat{j} + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$ intersect or not. If intersecting, find their-point of intersection.

20. Find λ so that the four points with position vectors -j+ k, 2i-j-k, i+ λ j+ k and 3i+3k are coplanar. OR Find the point on line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1, 2, 3)

21. Solve the differential equations $(\tan^{-1} y - x)dy = (1+y^2)dx$

22. Show that the differential equation $x \cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$ is homogeneous and solve it.

OR Show that the differential equation $xdy - ydx = \sqrt{x^2 + y^2}\, dx$ is homogeneous and solve it.

SECTION C

23. Evaluate: $\int_{-1}^{3/2} |x \sin \pi x|\, dx$ OR $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$

24. There are 3 bags each containing 5 white balls and 3 black balls, 2 bags each containing 2 white balls and 4 black balls and 3 bags each containing 4 white balls and 4 black balls. A white ball is drawn at random. Find the probability that this white ball is from the bag of first group.

25. If A= $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ and A^2-nA-mI=0, find m and n. Hence find A^{-1}.

26. Using integration find the area bounded by the line x+2y = 2, y-x = 1 and 2x+y = 7

27. Find the equation of line through the point (3, 4) which cuts the 1st quadrant a triangle of minimum area. OR A cylindrical container with a capacity of 20 cubic feet is to be produced. The top and bottom of the container are to be made of a material that costs Rs.6 per square foot while the side of the container is made of material costing Rs.3 per square foot. Find the dimension that will minimize the total cost.

28. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

29. Find image of the point (1,0,2) in the line passing through (2,1,3) and perpendicular to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$

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