



**ACADEMY OF SCIENCE,
BILASPUR , CHHATTISGARH
(IIT,CPMT,AIEEE, STATE PET,PMT COACHING
INSTITUTE) Contact no.-0971328199,09826819224
CLASS-XII
GUESS PAPER
MATHEMATICS**

Time-3 hr.

Marks-100

General Instructions:

- (i) All questions are compulsory.
- (ii) The question paper consists of 29 questions divided into four sections A,B,C and section A comprises of 10 questions of 01 mark each, Section B comprises of 12 questions of 04 marks each and Section C comprises of 07 questions of 6 marks each.
- (iii) All questions in Section A are to be answered in one word , one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice. However ,Internal choice has been provided in 04 questions of 4 marks each and 02 questions of 6 marks each. You have to attempt only one of the alternatives in all such questions .
- (v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Q.1 Show that the function $f:R \rightarrow R$ defined as $f(x) = 3x+4$ is one-one and onto .

Q.2 Write the principal value of
 $\tan^{-1}(1) + \cos^{-1}(-1/2) + \sin^{-1}(-1/2)$

Q.3 If

$$A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

Then $A + A' = I$, if the value of α is given by ?

Q.4 Given a matrix

$$A = \begin{bmatrix} 3 & -2 \\ 5 & 6 \end{bmatrix}$$

Find matrix kA where $k=-3$

Q.5 Find the point where the line $y=x+1$ is a tangent to the curve $y^2=4x$.

Q.6 Find the angle between the vectors $i-2j+3k$ and $3i-2j+k$.

Q.7 The Cartesian equation of a line is $(x-5)/3 = (y+4)/7 = (z-6)/2$ write its vector form.

Q.8 Write the value of $\int (\sin^2 x - \cos^2 x) / \sin^2 x \cdot \cos^2 x \, dx$

Q.9 Find a vector in the direction of $5i - j + 2k$ which has magnitude 8 units.

Q.10 Let A be a non singular square matrix of order 3×3 . Then $|\text{adj } A|$ is equal to ?

SECTION-B

Q.11 Let $f: \mathbb{N} \rightarrow \mathbb{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: \mathbb{N} \rightarrow S$, where S is the range of f , is invertible. Find the inverse of f . **OR**

Let $A = \mathbb{N} \times \mathbb{N}$ and $*$ be a binary operation on A defined by $(a,b) * (c,d) = (a+c, b+d)$. Show that $*$ is commutative and associative. Find the identity element for $*$ on A , if any.

Q.12 Prove that $\tan^{-1}(63/16) = \sin^{-1}(5/13) + \cos^{-1}(3/5)$

Q.13 By using elementary operations, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

OR

If x, y, z are different and

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0 \text{ then show that } 1+xyz=0$$

Q.14 Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

Q.15 If $y = (x)^{\sin x} + (\sin x)^x$, Find dy/dx . **OR**
Differentiate

$\tan^{-1} \left\{ \frac{(\sqrt{1+x^2} - \sqrt{1-x^2})}{(\sqrt{1+x^2} + \sqrt{1-x^2})} \right\}$ with respect to $\sin^{-1} \left[\frac{2x}{1+x^2} \right]$

Q.16 Evaluate : $\int \sqrt{\tan \theta} \, d\theta$

Q.17 Water is leaking from a conical funnel at the rate of $5 \text{ cm}^3/\text{sec}$. If the radius of the base of funnel is 5 cm and height 10 cm, find the rate at which is the water level is dropping when it is 2.5 cm from the top.

Q.18 Evaluate $\int_0^2 (x^2 + x + 2) dx$ as limit of sums.

Q.19 Evaluate : $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

Q.20 Find the coordinates of the foot of the perpendicular drawn from the point A(1,8,4) to the line joining the points B(0,-1,3) and C(2,-3,-1).

Q.21 Express the vector $\vec{a} = 5i - 2j + 5k$ as sum of two vectors such that one is parallel to the vector $\vec{b} = 3i + k$ and the other is perpendicular to \vec{b}

Q.22 A factory has 3 machines X, Y and Z producing 1000, 2000 and 3000 bolts per day respectively. The machine X produces 1% defective bolts, Y produces 1.5% and Z produces 2% defective bolts. At the end of a day, a bolt is drawn at random and is found defective. What is the probability that this defective bolt has been produced by the machine X ? **OR**

A bag contains 6 red and 5 blue and another bag contains 5 red and 8 blue balls. A ball is drawn from the first bag and without noticing its colour is put in the second bag. A ball is then drawn from the second bag. Find the probability that the ball drawn is blue in colour.

SECTION-C

Q.23 Find the matrix A satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

OR

Using matrices, solve the following system of equations :

$$2x + y - 3z = 13$$

$$x + y - z = 6$$

$$2x - y + 4z = -12$$

Q.24 A rectangle is inscribed in a semicircle of radius r with one of its sides on diameter of semicircle. Find the dimensions of the rectangle so that its area is maximum. Find the area also.

OR

Show that the height of cylinder of maximum volume that can be inscribed in sphere of radius r is $\frac{2r}{\sqrt{3}}$.

Q.25 Find the area of the following region :

$$\{(x,y) : x^2 + y^2 \leq 2ax, y^2 \geq ax, x \geq 0, y \geq 0\} \quad \text{OR}$$

Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

Q.26 Solve the following differential equation :

$$2xydx + (x^2 + 2y^2) dy = 0$$

Q.27 Find the Cartesian as well as the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$ which are at unit distance from origin.

Q.28 A random variable X has the following probability distribution values of X

X =	0	1	2	3	4	5	6	7
P(X) =	0	K	2K	2K	3K	K ²	2K ²	7K ² + K

Find each of the following :

- K
- $P(X < 6)$
- $P(X \geq 6)$
- $P(0 < X < 5)$

Q.29 A retired person has Rs. 70,000 to invest and two types of bonds are available in the market for investment. First type of bonds yields an annual income of 8% on the amount invested and the second type of bonds yields 10% per annum. As per norms, he has to invest a minimum of Rs. 10,000 in the first type and not more than Rs. 30,000 in the second type. How should he plan his investment, so as to get maximum return, after one year of investment ?

OR

A firm manufactures two types of products A and B sells them at a profit of Rs. 5 per unit of type A and Rs. 3 per unit of type B. Each product is processed on two machines M_1 and M_2 . One unit of type A requires one minute of processing time on M_1 and two minutes of processing time on M_2 ; whereas one unit of type B requires one minute of processing time on M_1 and one minute on M_2 . Machines M_1 and M_2 are respectively available for almost 5 hours and 6 hours in a day. Find out how many units of each type of product should the firm produce a day in order to maximize the profit. Solve the problem graphically.

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