CBSE DELHI ANNUAL EXAMINATION 2015

MATHEMATICS

Class XII

Set 1

Max Time 3 Hrs **Max Marks 100**

General instructions:

1- All questions are compulsory.
2- This question paper consists of twenty nine questions divided into three sections A, B, C. Section A comprises of six questions of 1 mark each, Section B comprises of thirteen questions of 4 marks each and Section C comprises of seven questions of 6 marks each.
3- There is no overall choice. However, internal choice has been provided in some questions.
Note for the students: All the 3 sets have all questions same with order changed.

Section A

Question numbers 1 to 6 carry 1 mark each.

1. If $\vec{a} = 7\hat{\imath} + \hat{\jmath} - 4\hat{k}$ and $\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$, then find the projection of $\vec{a}$ on $\vec{b}$.
2. Find λ, if the vectors $\vec{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$ and $\vec{c} = \lambda\hat{\jmath} + 3\hat{k}$ are coplanar.
3. If a line makes angles $90^0, 60^0$ $and\ \theta$ with $x, y\ and\ z - axis$ respectively, where θ is acute, then find θ.
4. Write the element a_{23} of a 3 X 3 matrix $A = (a_{ij})$ whose elements a_{ij} are given by $a_{ij} = \frac{|i-j|}{2}$.
5. Find the differential equation representing the family of curves $v = \frac{A}{r} + B, where$ A and B are arbitrary constants.
6. Find the integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$.

Section B

Question numbers 7 to 19 carry 4 marks each.

7. Let $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$ find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = 0$.

Or

If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$.

8. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinants find the value of $f(2x) - f(x)$.

9. Find : $\int \frac{dx}{\sin x + \sin 2x}$ **OR** Integrate w.r.t. x $\frac{x^2-3x+1}{\sqrt{1-x^2}}$

10. Evaluate $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 dx$

11. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, find the probability of one of them being red and another black.

OR

An unbiased coin is tossed 4 times. Find the mean and variance of the number of heads obtained.

12. If $\vec{r} = x\,\hat{i} + y\hat{j} + z\hat{k}$, find $(\vec{r} \times \hat{i}).(\vec{r} \times \hat{j}) + xy$.

13. Find the distance between the point (-1,-5,-10) and the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$.

14. If $\sin[\cot^{-1}(x+1)] = \cos(\tan^{-1} x)$, then find x.

OR

If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then find x.

15. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1+x^2}}{\sqrt{1+x^2}-\sqrt{1+x^2}}\right), x^2 \leq 1$, then find $\frac{dy}{dx}$.

16. If $x = a\cos\theta + b\sin\theta, y = a\sin\theta - b\cos\theta$, show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

17. The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing when the side of the triangle is 20 cm ?

18. Find : $\int (x+3)\sqrt{3-4x-x^2}\,dx$.

19. Three schools A, B and C organized a mela for collecting fund for helping the rehabilitation to raise money for an orphanage, of flood victims. They sold hand made fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold are given below:

Article \ School	A	B	C
Hand fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Find the funds collected by each school separated by selling the given adjacent articles. Also find the total funds collected for the purpose.
Write one value generated by the above situation.

Section C

Question numbers 20 to 26 carry 6 marks each.

20. Let N denotes the set of all natural numbers and R be the relation on N X N defined by (a, b) R (c, d) if ad(b+c) = bc(a+d). Prove that R is an equivalence relation.

21. Using integration find the area of the triangle formed by positive x-axis and tangent and normal to the circles $x^2 + y^2 = 4 \ at \ (1, \sqrt{3})$.

OR

Evaluate $\int_1^3 (e^{2-3x} + x^2 + 1)dx \ \ as \ a$ limit of a sum.

22. Solve the differential equation : $(\tan^{-1} y - x)dy = (1 + y^2)dx$.

OR

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ given that y = 1, when x = 0.

23. If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find the value of k and hence find the equation of the plane containing these lines.

24. If A and B are two independent events such that P($\bar{A} \cap B$) = $\frac{2}{15}$ and P($A \cap \bar{B}$) = $\frac{1}{6}$, then find P(A) and P(B).

25. Find the local maxima and local minima, of the function f(x) = $\sin x - \cos x, 0 < x < 2\pi$. Also find the local maximum and local minimum values.

26. Find graphically, the maximum value of z = 2x + 5y, subject to constraints given below:

$2x + 4y \leq 8$

$3x + y \leq 6$

$x + y \leq 4$

$x \geq 0, y \geq 0$

Answer Key and hints

Section A

1. $\frac{4\sqrt{66}}{231}$ 2. 7 3. 30^0 4. $\frac{1}{2}$ 5. $2\frac{dv}{dr} + r^2\frac{d^2v}{dr^2} = 0$ 6. $e^{2\sqrt{x}}$

Section B

7. $\begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$, X = $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$ 7(OR) $\frac{1}{3}\begin{bmatrix} -7 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

8. f(x) = a$(x + a)^2$ after solving the det. Now $f(2x) - f(x) = a(2x + a)^2 - a(x + a)^2 = ax(3x + 2a)$

9. $\int \frac{dx}{sinx+2sinx.\cos x} = \int \frac{dx}{sinx(1+2cosx)} = -\int \frac{dt}{(1-t^2)(1+2t)} = -\frac{1}{6}\log|1 - t| - \frac{1}{2}\log|1 + t| +$ $23\log 1+2t+c$, where $t=\cos x$ 10. $x21-x2-3x1-x2+11-x2\ dx=-$ $\int \frac{-x^2}{\sqrt{1-x^2}}dx - 3\sqrt{1 - x^2} + \sin^{-1} x = -\int \frac{1-x^2-1}{\sqrt{1-x^2}}dx - 3\sqrt{1 - x^2} + \sin^{-1} x = -\int \sqrt{1 - x^2}dx +$ $11-x2dx-31-x2+\sin-1x=-x21-x2+32\sin-1x-31-x2+c$ 10.

Section C

20. For the relation R defined by (a, b) R (c, d) if ad(b+c) = bc(a+d) on N X N.

Reflexivity: Let (a, b) ∈ N X N. (a, b) R (a, b) as ab(b+a) = ba(a+b) b + a (Addition and Multiplication are commutative on N) ⇒ (a, b) R (a, b) ∀ (a, b)∈ N X N.

∴ R is reflexive relation on N X N.

Symmetry: Let (a, b) R (c, d) ⇒ ad(b+c) = bc(a+d) ∀ a, b, c, d ∈ N

⇒ cb(d+a) = da(c+b) ∀ a, b, c, d ∈ N (Addition is commutative on N)

⇒ (c, d) R (a, b) ∀ a, b, c, d ∈ N.

∴ R is symmetric relation on N X N.

Transitivity: Let (a, b) R (c, d) ⇒ ad(b+c) = bc(a+d) ∀ a, b, c, d ∈ N ... (i)

& (c, d) R (e, f) ⇒ cf(d+e) = de(c+f) ∀ c, d, e, f ∈ N ... (ii)

Multiplying (i) by ef and (ii) by ab and then adding we get

efad(b+c) + abcf(d+e)= ef bc(a+d) + abde(c+f)

$abdef + acdef + abcdf + abcef = abcef + bcdef + abcde + abdef$

$\Rightarrow acdef + abcdf = bcdef + abcde$ (Multiplication is commutative on N)

Divide by cd both sides

$aef + abf = bef + abe$

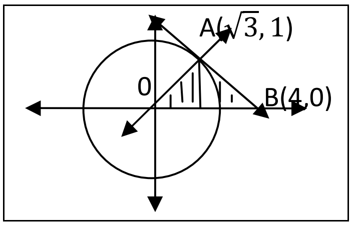
$\Rightarrow af(b+e) = be(a+f)$

⇒ (a, b) R (e, f) ∀ a, b, c, d, e, f ∈ N

∴ R is transitive relation on N X N.

∵ R is reflexive, symmetric and transitive so R is equivalence relation.

21. Tangent AB $x + \sqrt{3}y = 4$ and Normal OA $y = \sqrt{3}x$



Required area = $\int_0^{\sqrt{3}} y_{OA}dx + \int_{\sqrt{3}}^{4} y_{OB}\,dx =$

$\int_0^{\sqrt{3}} \sqrt{3}x dx + \frac{1}{\sqrt{3}}\int_{\sqrt{3}}^{4}(4-x)dx = \frac{14\sqrt{3}}{3} - 4\ sq.units$

21(OR) $-\frac{e^{-7}}{3}+\frac{e^{-1}}{3}+\frac{32}{3}$ 22. $\left(x = (\tan^{-1}y - 1) + Ce^{-\tan^{-1}y}\right)$ using $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$

23. Equating x coordinate μ = 2λ-2, equating z coordinate 4λ+1=μ solving these λ= $-\frac{3}{2}$ and μ= -5

equating y coordinate k= $\frac{9}{2}$; Equation of plane is given by $\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow -5x + 2y + z + 6 = 0$ 24. P($\bar{A} \cap B$)= $\frac{2}{15}$ ⇒P($\bar{A}$).P(B)= (1- P(A))P(B)= $\frac{2}{15}$, (1-P(B))P(A)= $\frac{1}{6}$

Taking P(A)=x and P(B)=y equation will be $30y^2 - 29y + 4 = 0 \Rightarrow$ y= $\frac{4}{5}, \frac{1}{6}$

When P(B)=$\frac{1}{6}$, $P(A)=\frac{1}{5}$; when P(B)=$\frac{4}{5}$, P(A)=$\frac{5}{6}$ 25. From $f'(x)=0 \Rightarrow$ x=$\frac{3\pi}{4}, \frac{7\pi}{4}$, $f''(x) = -\sin x + \cos x$ Max at x= $\frac{3\pi}{4}$ $is\ \sqrt{2}\ and$ Min at x= $\frac{7\pi}{4}$ is -$\sqrt{2}$

26. Feasible region is OABCO

Solving B($\frac{8}{5}, \frac{6}{5}$)

$Z_o = 0,\ Z_A = 4, Z_C = 10, Z_B = \frac{46}{5}$

So Max at B($\frac{8}{5}, \frac{6}{5}$) is $Z_B = \frac{46}{5}$