CBSE MATHEMATICS GUESS/SAMPLE PAPER

Time allowed: 3 hours Maximum Marks: 100

(Candidates are allowed additional 15 minutes for only reading the paper.
*They must **not** start writing during this time)*

General Instructions:

(i) All questions are compulsory.
(ii) The question paper consists of **29** questions divided into three sections A, B and C. Section A comprises of **10** questions of **one mark** each, section B comprises of **12** questions of **four marks** each and section C Compromises of **7** questions of **six marks** each.
(iii) All questions in Section A are to be answered in one word, one sentence Or as per the exact requirement of the question.
(iv) There is no overall choice.
(v) Use of calculator is **not** permitted.

SECTION A

Questions number 1 to 10 carry 1 mark each

1. Is the function $f: N \to N$, defined by $f(n) = 2n + 5$, is invertible? Give reason.

2. Find the value of $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(\sin\frac{2\pi}{3}\right)\right]$

3. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, f(x) = x^2 - 2x - 3,$ show that $f(A) = O$.

4. Let R be the set of all real numbers. Define a relation T on R by $T = \{(a,b) : 1 + ab > 0\}$. Is T an equivalence relation?

5. Evaluate $\begin{vmatrix} \tan\beta & \cos ec\beta \\ \sin\beta & \cot\beta \end{vmatrix}$.

6. Evaluate $\int (x\tan x + \log|\sec x|)dx$

7. If $y = e^{a^x}$, find $\frac{dy}{dx}$

8. Find $a.c$ if $|a| = 6$, $|c| = 2$ and $|a \times c| = 5$

9. Find the direction cosine of the line equally inclined to the positive direction of the axes.

10. Find the angle between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-4}{6}$ and $x + 1 = \frac{y+2}{2} = \frac{z-4}{2}$

SECTION B

Questions number 11 to 22 carry 4 marks each

11. Let $f: R \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5,$ where R is the set of positive numbers including zero. Show that f is invertible and find f^{-1}

12. Solve for x: $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$ OR Show that $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$

13. Evaluate: $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix}$

14. Let $f(x)=\dfrac{1-\tan x}{4x-\pi}, x\neq\dfrac{\pi}{4}, x\in\left[0,\dfrac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0,\dfrac{\pi}{2}\right]$, find $f\left(\dfrac{\pi}{4}\right)$.

15. If $y=\left(x+\sqrt{1+x^2}\right)^n$, then show that $(1+x^2)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-n^2y=0$.

16. Find the intervals in which the function $f(x)$ is increasing or decreasing where $f(x)=xe^{x(1-x)}$ OR

Find the normal at any point 'θ' on the curve $x=a(\cos\theta+\theta\sin\theta)$ $y=a(\sin\theta-\theta\cos\theta)$

17. Evaluate: $\int\dfrac{2x^2+3}{(x^2-1)(x^2+4)}dx$ OR $\int_{\pi/4}^{3\pi/4}\dfrac{\delta}{\sin\delta}d\delta$

18. Find the differential equation representing the family of curves $y^2=2c(x+\sqrt{c})$ where $c>0$ is a parameter. Also find the order and degree.

19. Solve $\dfrac{dy}{dx}=1+y+y^2+x+xy+xy^2$

20. Using vector method show that if the mid-points of the consecutive sides of a quadrilateral are joined, the resulting figure is a parallelogram.

21. Find the vector equation of the plane passing through the points (3,-5,-1) and (-1,5,7) and parallel to the vector $3\hat{i}-\hat{j}+7\hat{k}$

22. Three students appear at an examination. The probabilities of their success are $\dfrac{1}{3}$, $\dfrac{1}{4}$ and $\dfrac{1}{5}$ respectively. Find the probability of success of at least two students.

SECTION C

Questions number 23 to 29 carry 6 marks each

23. Solve by matrix method $\dfrac{2}{x}+\dfrac{3}{y}+\dfrac{10}{z}=4$ $\dfrac{4}{x}+\dfrac{6}{y}+\dfrac{5}{z}=1$ $\dfrac{6}{x}+\dfrac{9}{y}+\dfrac{20}{z}=2$

24. A man 2 meters high walks at a uniform speed of 5 km/hr away from a lamp post 6 meters high. Find the rate at which the length of his shadow increases.

25. Using integration find the area of the region enclosed between the circles $x^2+y^2=4$ and $(x-2)^2+y^2=4$.

26. Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$ OR $\int_0^3 (2x^2+5)dx$

27. Find the length and the foot of the perpendicular from the point (1,1,2) to the plane $r.(2\hat{i}-2\hat{j}+4\hat{k})+5=0$.

28. A farmer has a supply of chemical fertilizer of type I which contains 10% Nitrogen and 5% Phosphoric acid and type II which contains 6% Nitrogen and 10% Phosphoric acid. After soil testing it was found that at least 14kg of Nitrogen and 14kg of Phosphoric acid is required for a good crop. The type I costs Rs. 2/kg and the type II cost Rs. 3/kg. How many kilograms of each fertilizer should be used to meet the requirement so that the cost be minimum?

29. A lot of 100 bulbs is known to contain 10 defective and 90 non-defective bulbs. If 8 bulbs are selected at random, what is the probability that
(i) there will be 3 defective and 5 non-defective bulbs
(ii) there will be atleast one defective bulb.

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